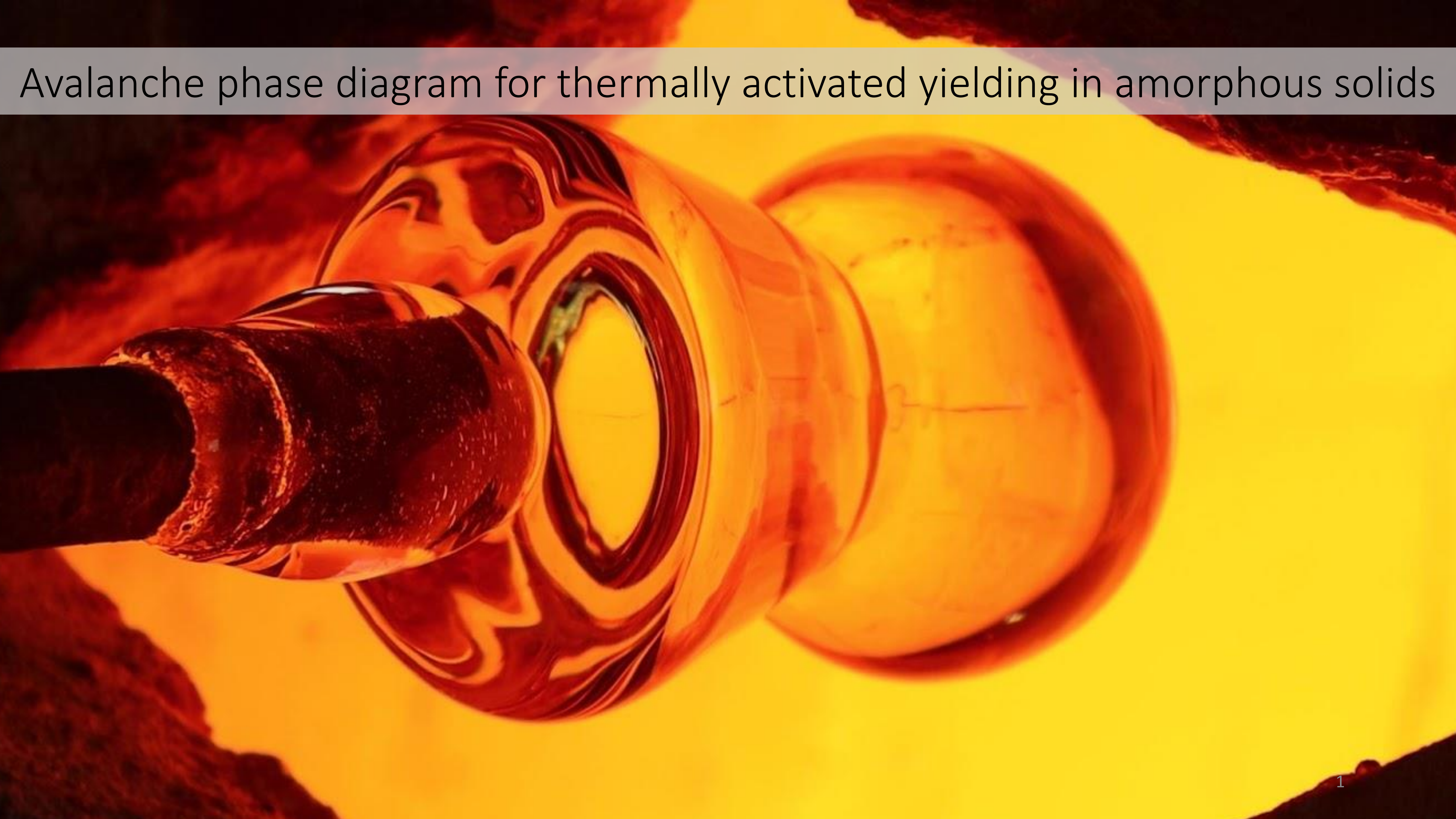
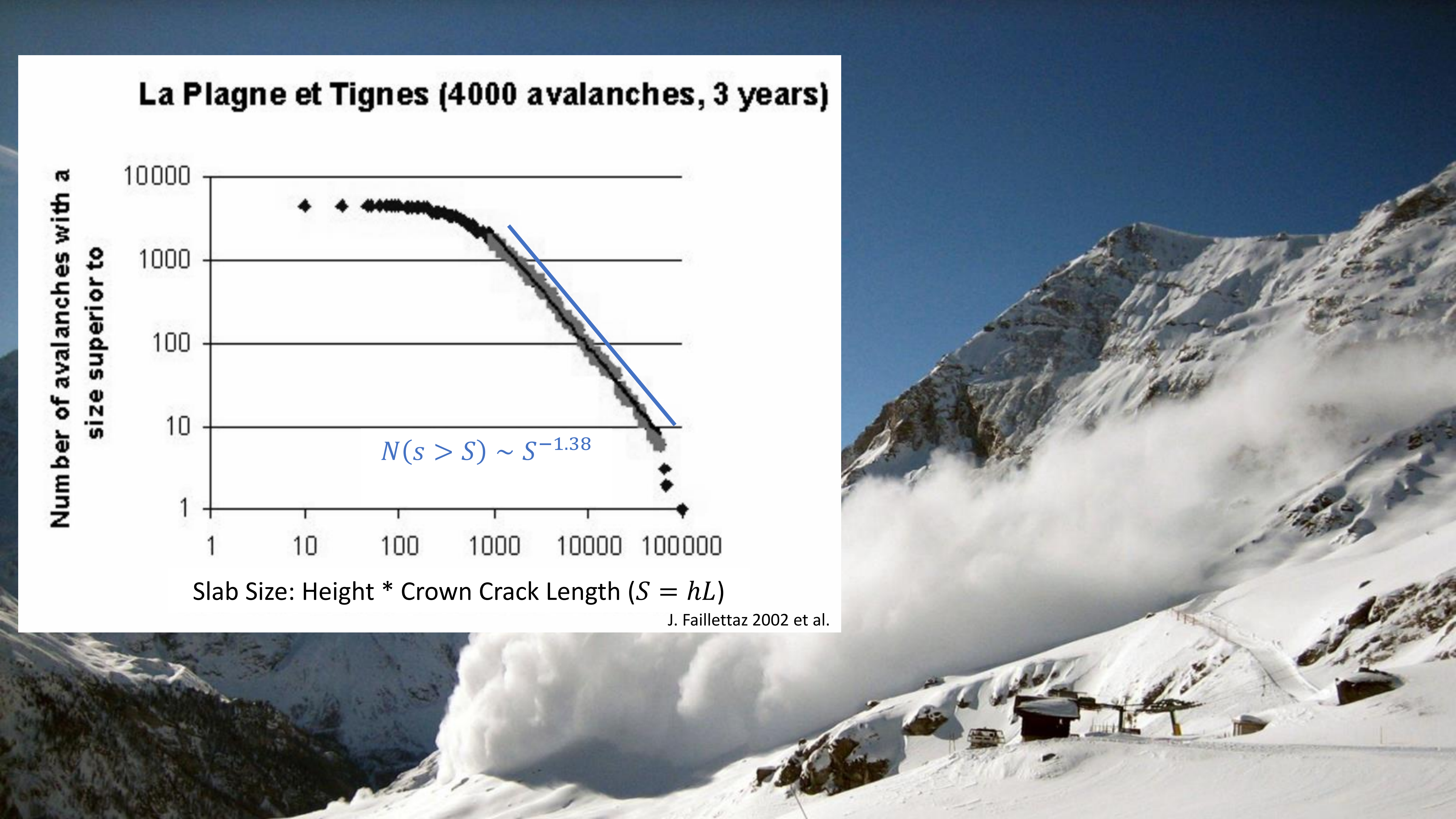
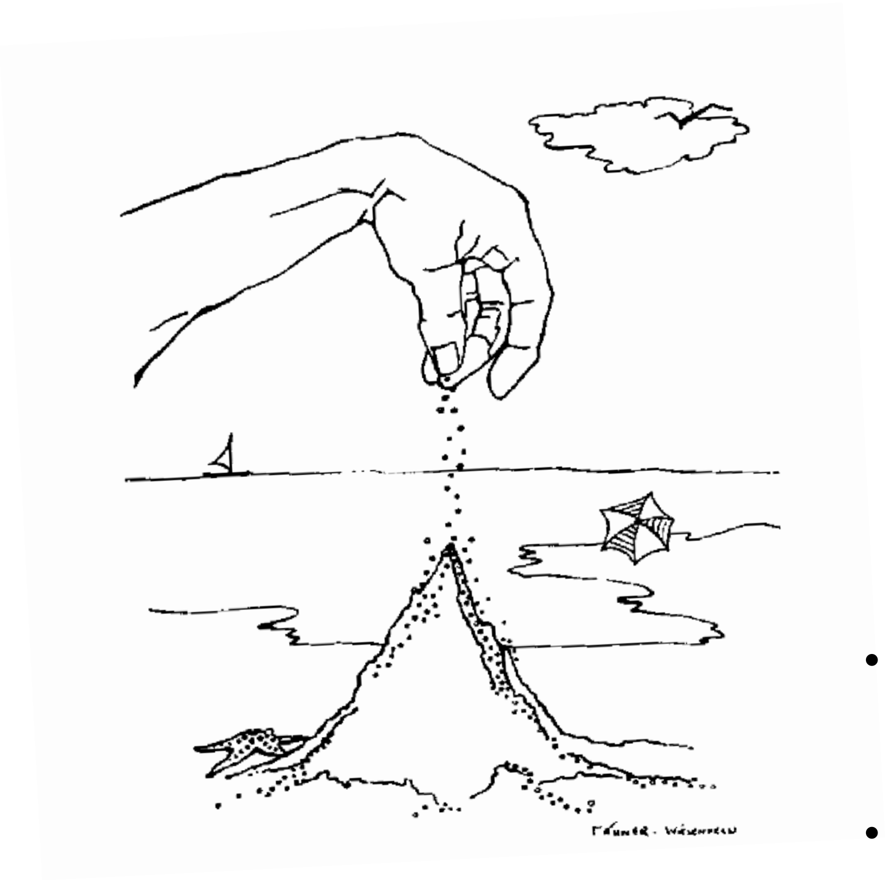
Avalanche phase diagram for thermally activated yielding in amorphous solids




University of British Columbia, Vancouver
Natural Sciences and Engineering Research Council of Canada
Conseil de recherches en sciences naturelles et en génie du Canada
Canada


La Plagne et Tignes (4000 avalanches, 3 years)
Number of avalanches with a size superior to
10000
1000
100
10
1
1
10
100
1000
10000
100000
$N(s > S) \sim S^{-1.38}$
Slab Size: Height * Crown Crack Length ($S = hL$)
J. Faillettaz 2002 et al.

Why power-laws? Self-organized criticality!



- Two ingredients:
  1. Loading: Grains added one grain at a time
  2. Avalanching: grains can fall, and knock into other grains, causing them to fall
- System self-tunes to a critical point
  - Power-law fluctuations or avalanches
  - Highly sensitive to input, single skier / grain of sand can create a huge avalanche (susceptibility goes to infinity)

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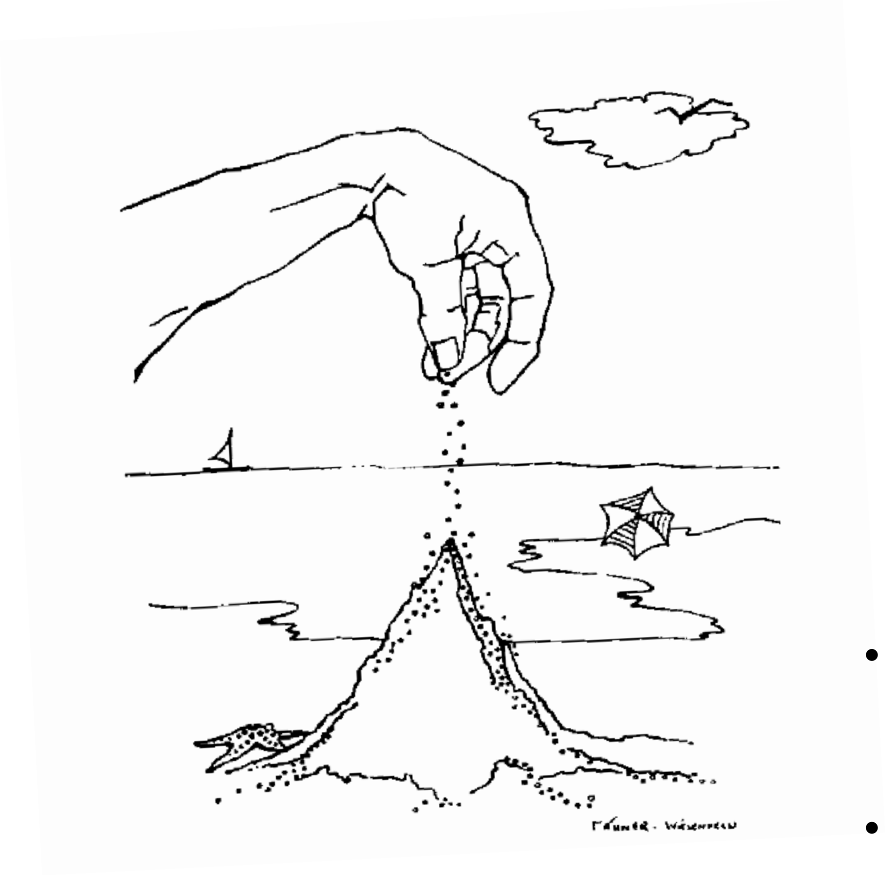

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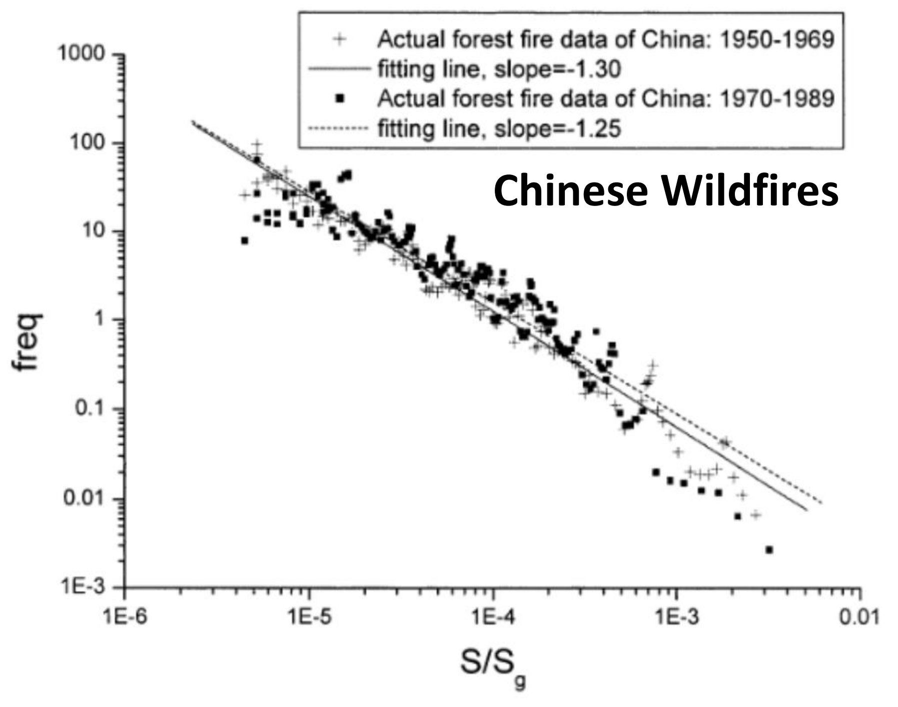
Avalanches are everywhere!



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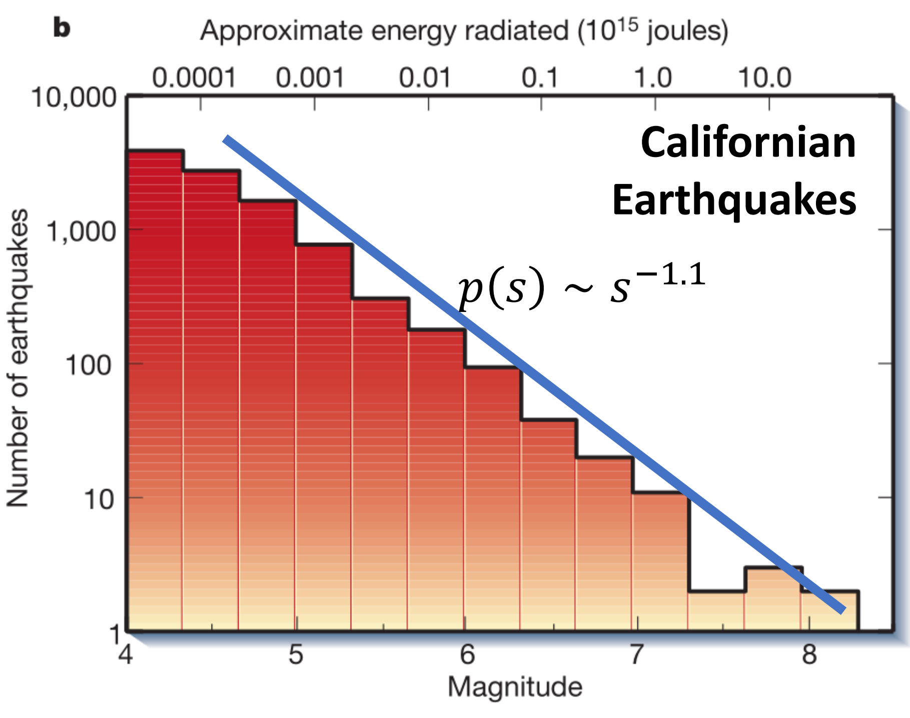

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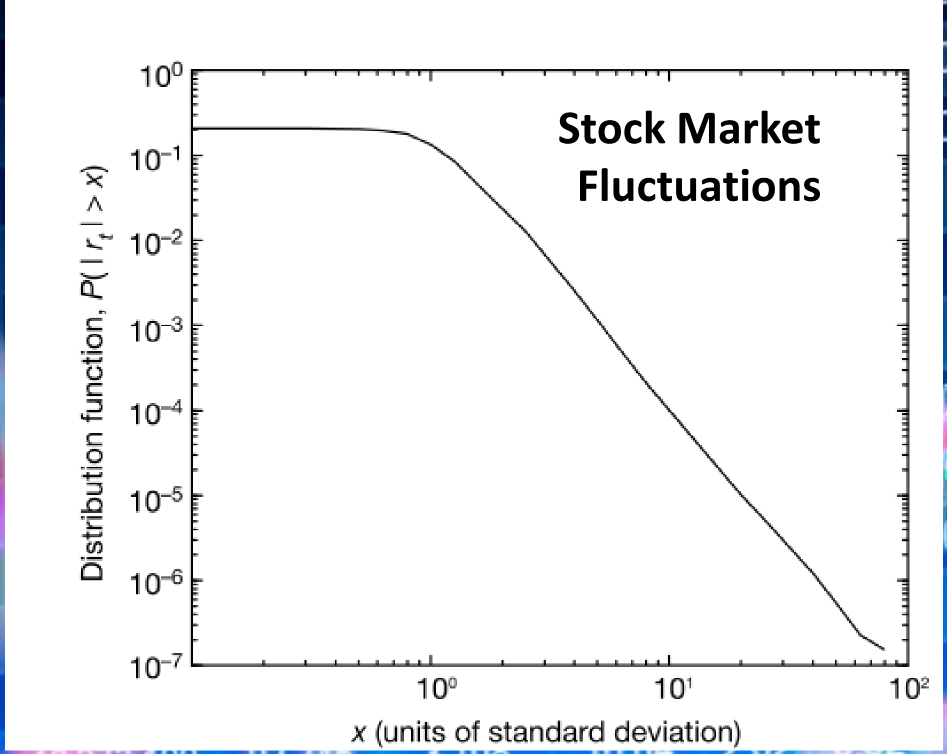

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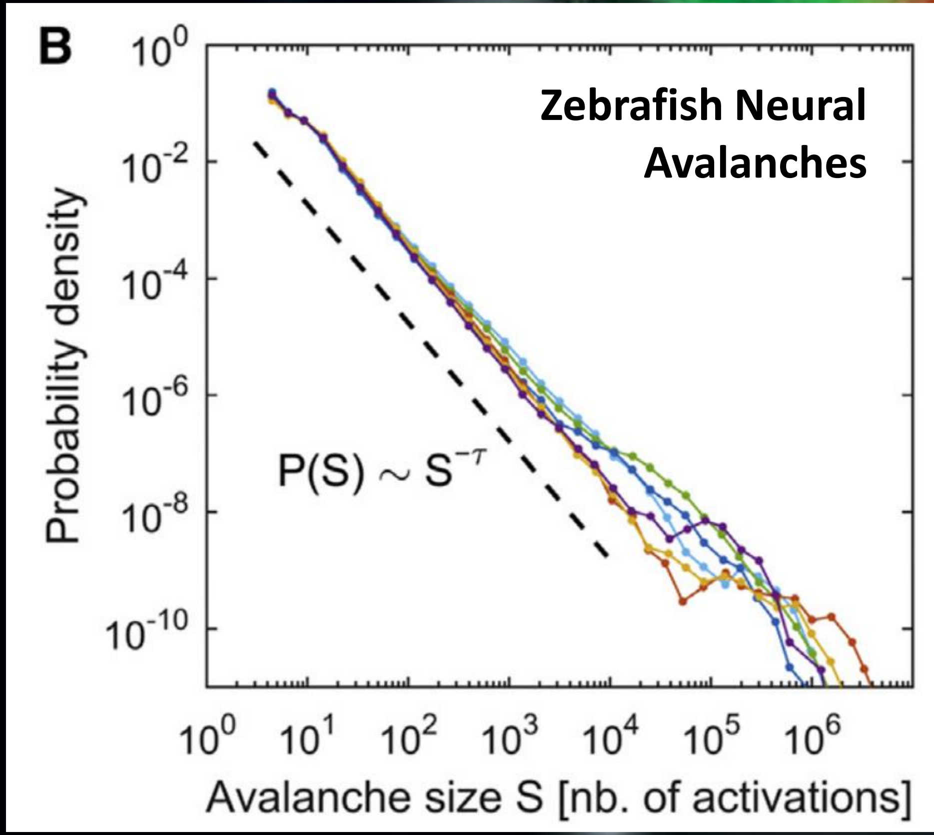
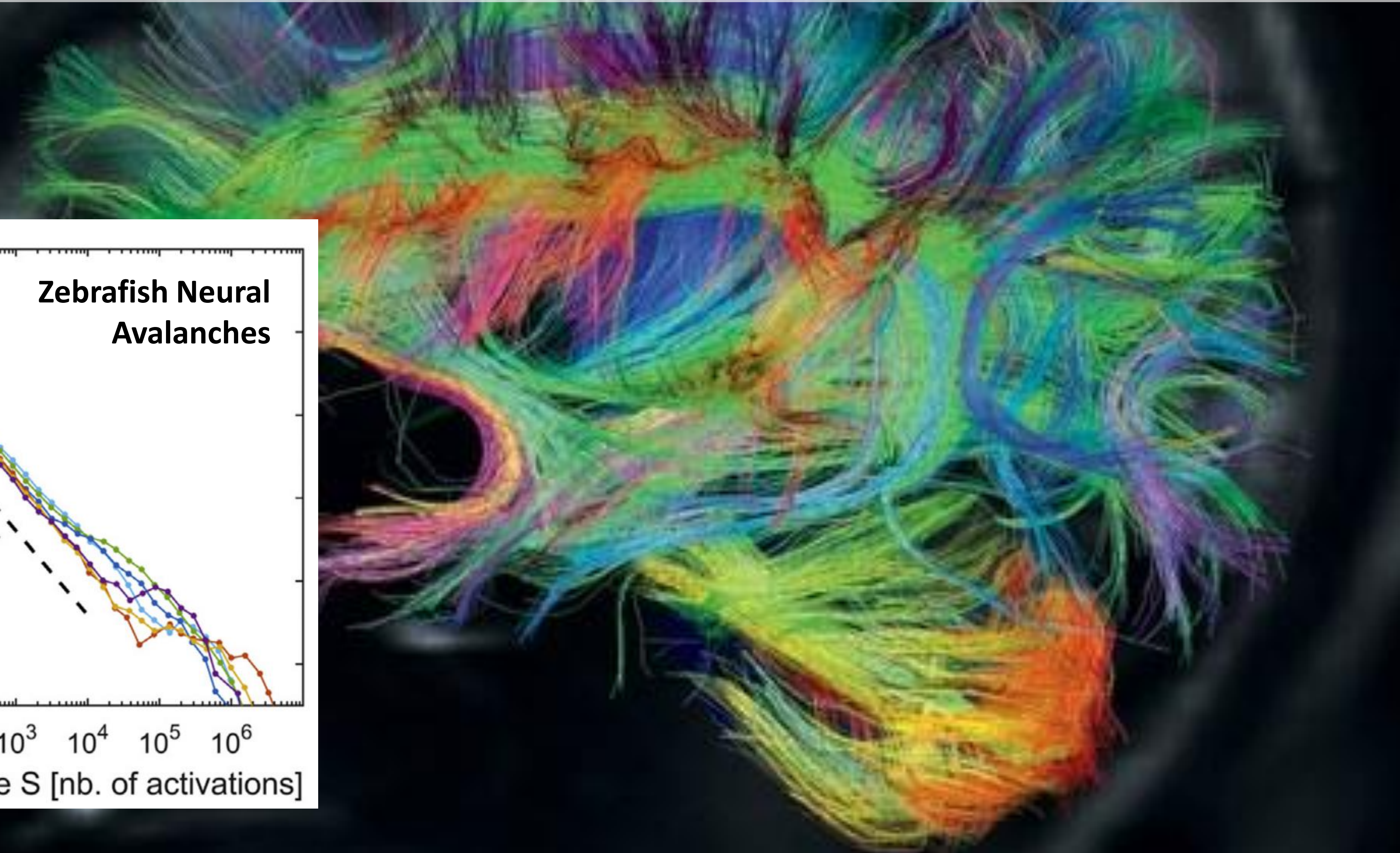
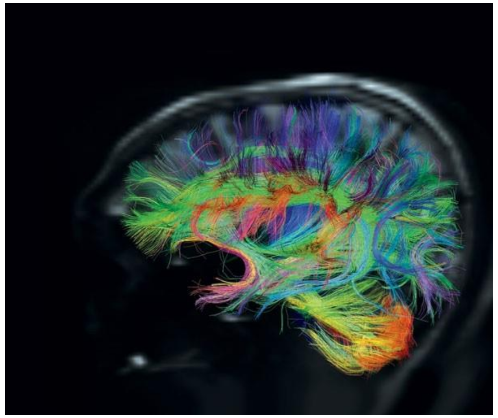
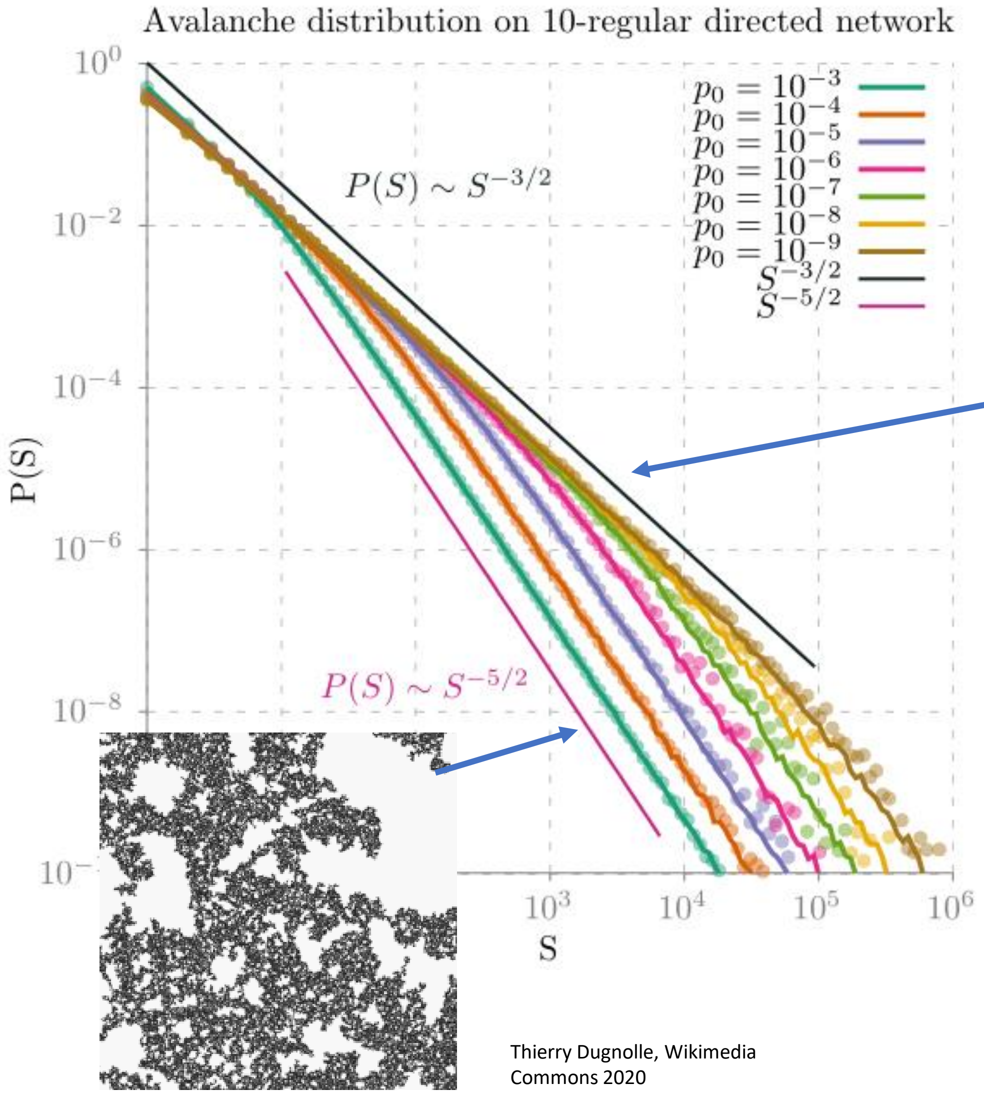


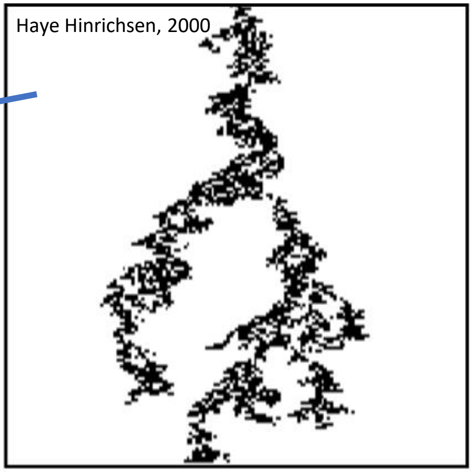


Can quickly driven avalanches be critical?

Driven neuronal avalanches can be critical







Haye Hinrichsen, 2000

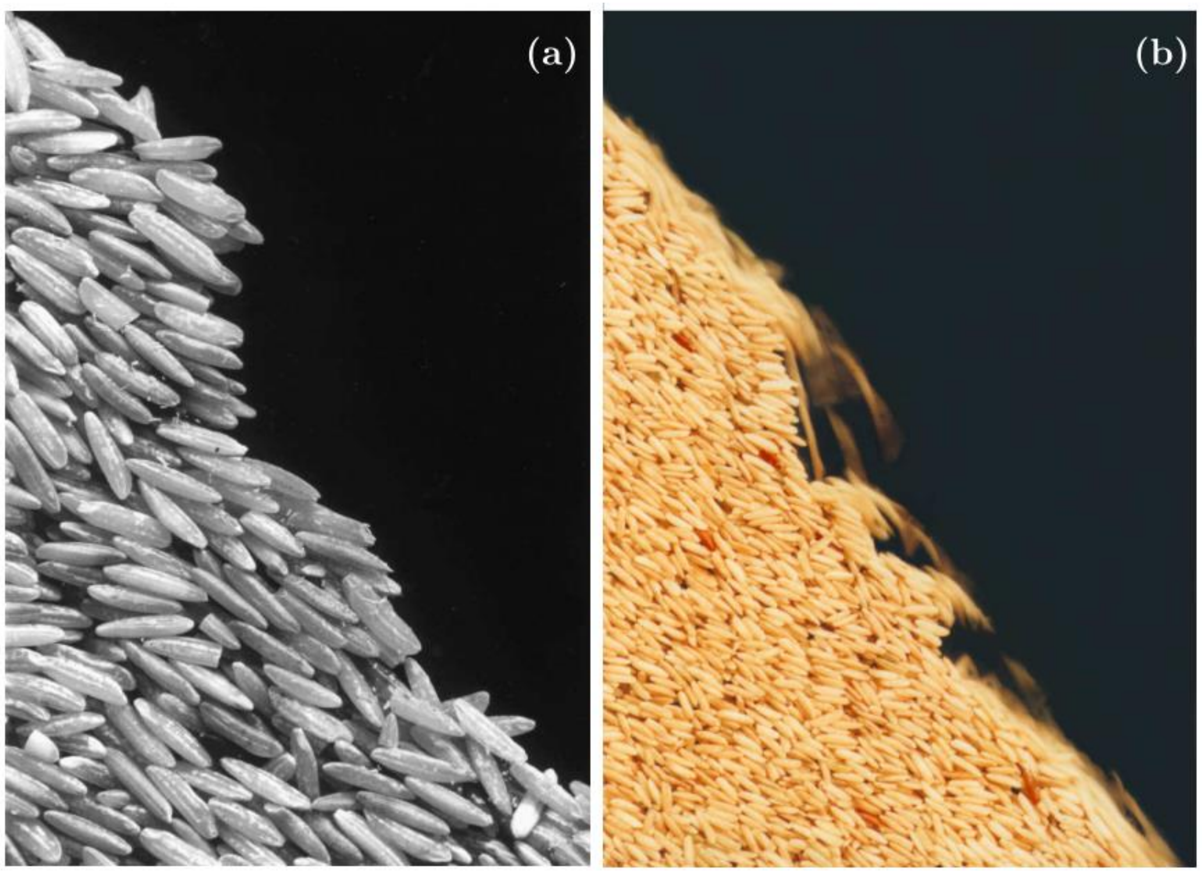
Thierry Dugnolle, Wikimedia Commons 2020

Daniel J. Korchinski, Javier G. Orlandi, Seung-Woo Son, and Jörn Davidsen
Phys. Rev. X **11**, 021059 – Published 17 June 2021

Driven rice avalanches are NOT critical

- Rice pile, critical for new grains at rate $\lambda < \lambda_c(L) \sim 1/\mathrm{L}^{(1+\mathrm{z}-\mathrm{D})\approx 0.2}$

Álvaro Corral and Maya Paczuski, PRL, 1999

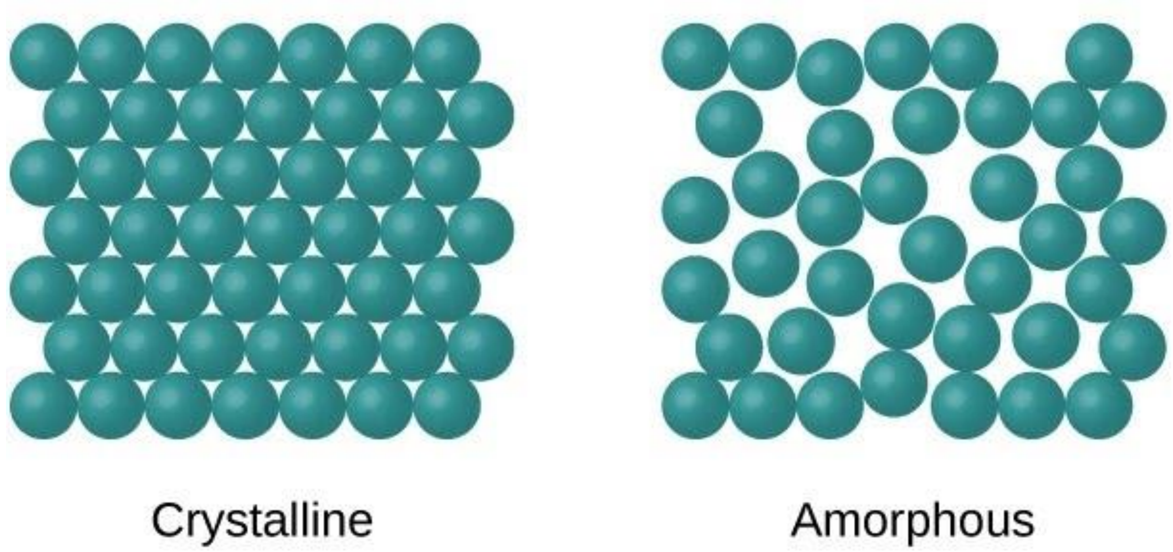


Kim Christensen, Nicholas R. Moloney, 2005

Objectives

1. Convince you self-organized criticality and avalanches are interesting
2. Introduce avalanches in amorphous solids with an “elastoplastic” model
3. Show you what happens when avalanches are also activated by temperature fluctuations

Amorphous yielding transition



Amorphous solid graphic: opentextbc.ca

Alexandre Nicolas et al. 2018. *Review of Modern Physics*

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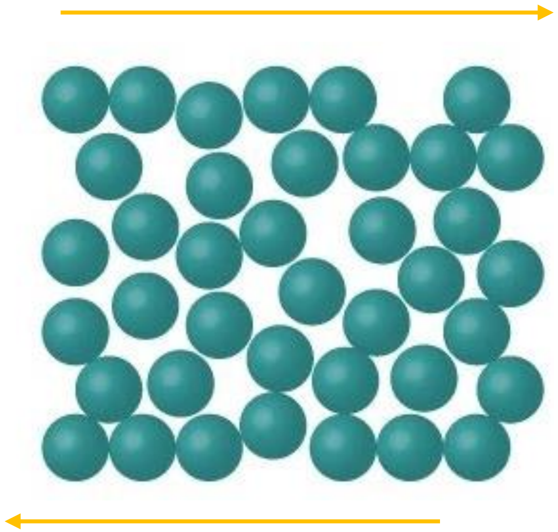

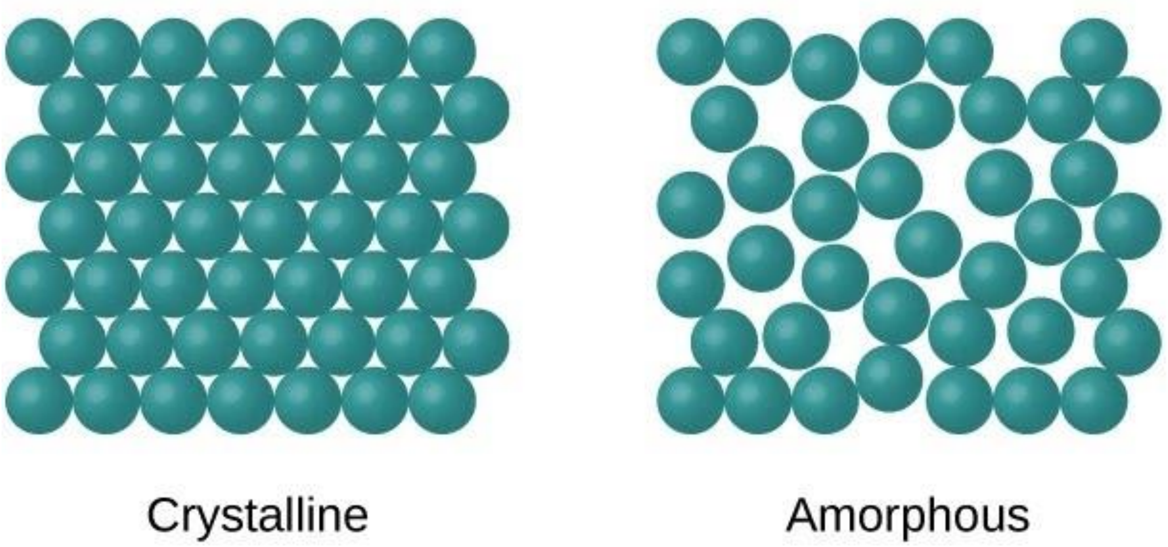

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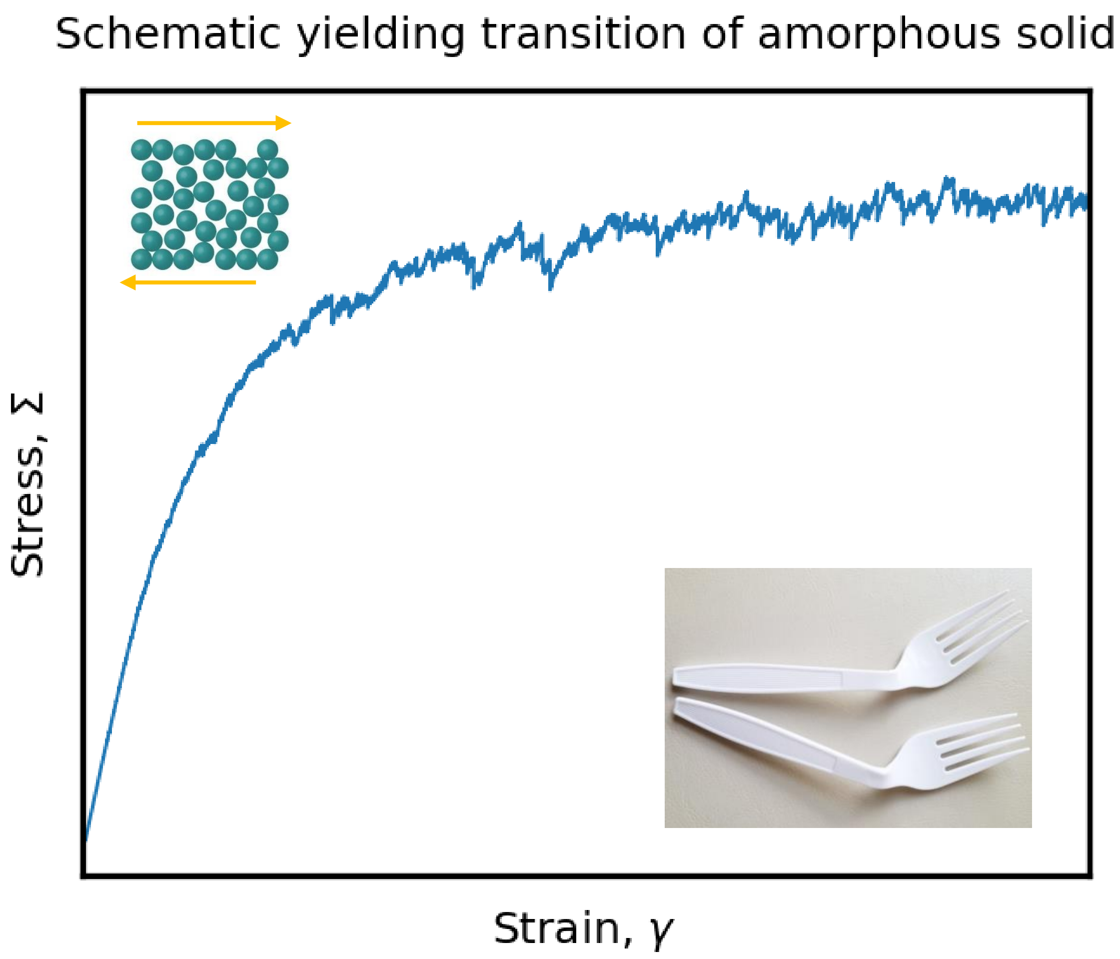

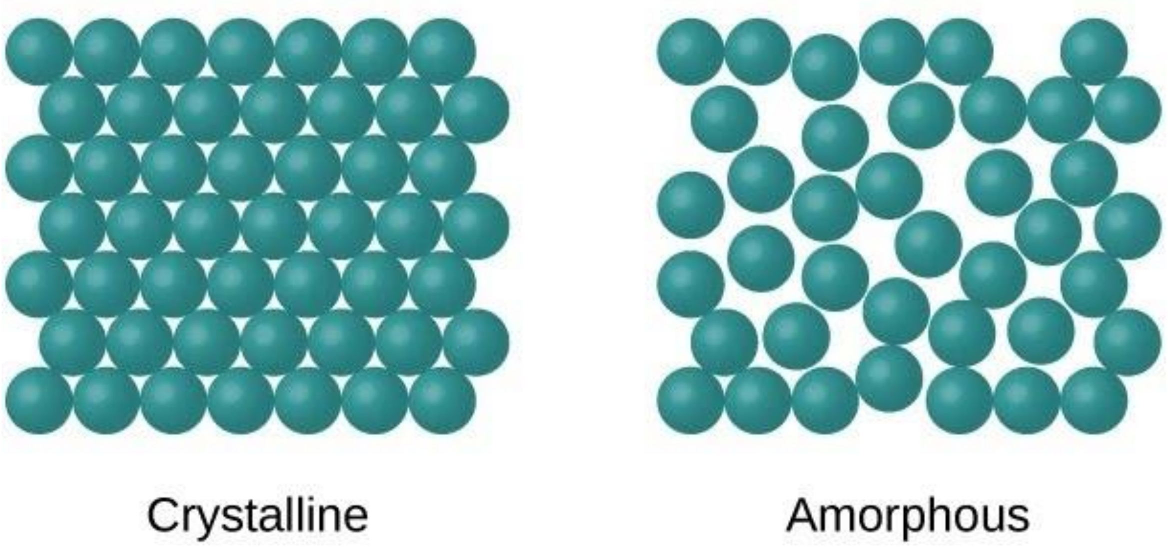

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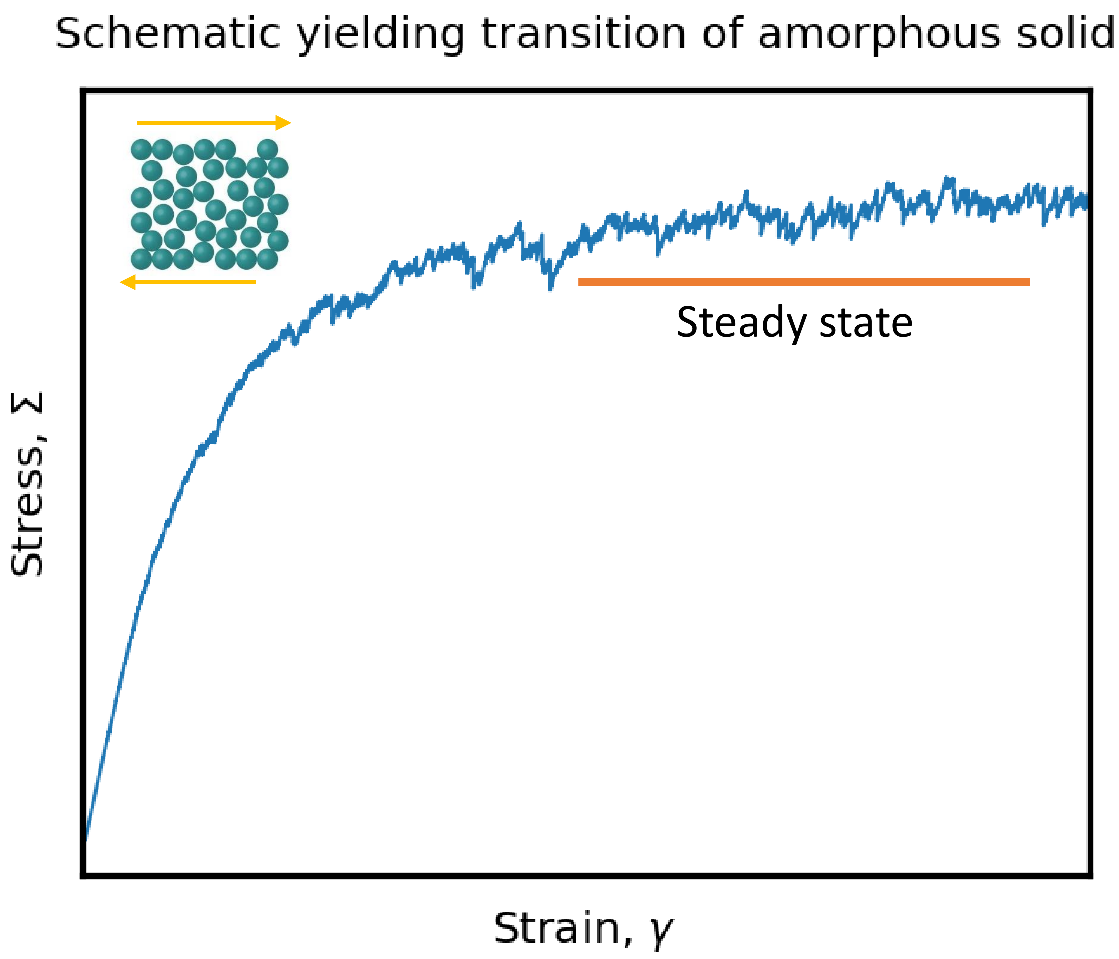

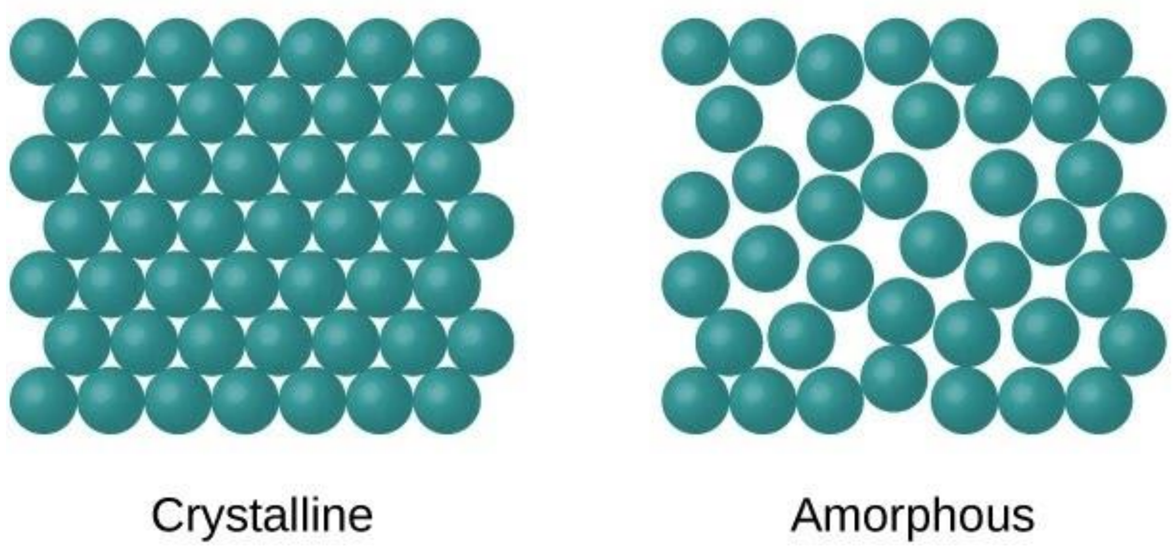

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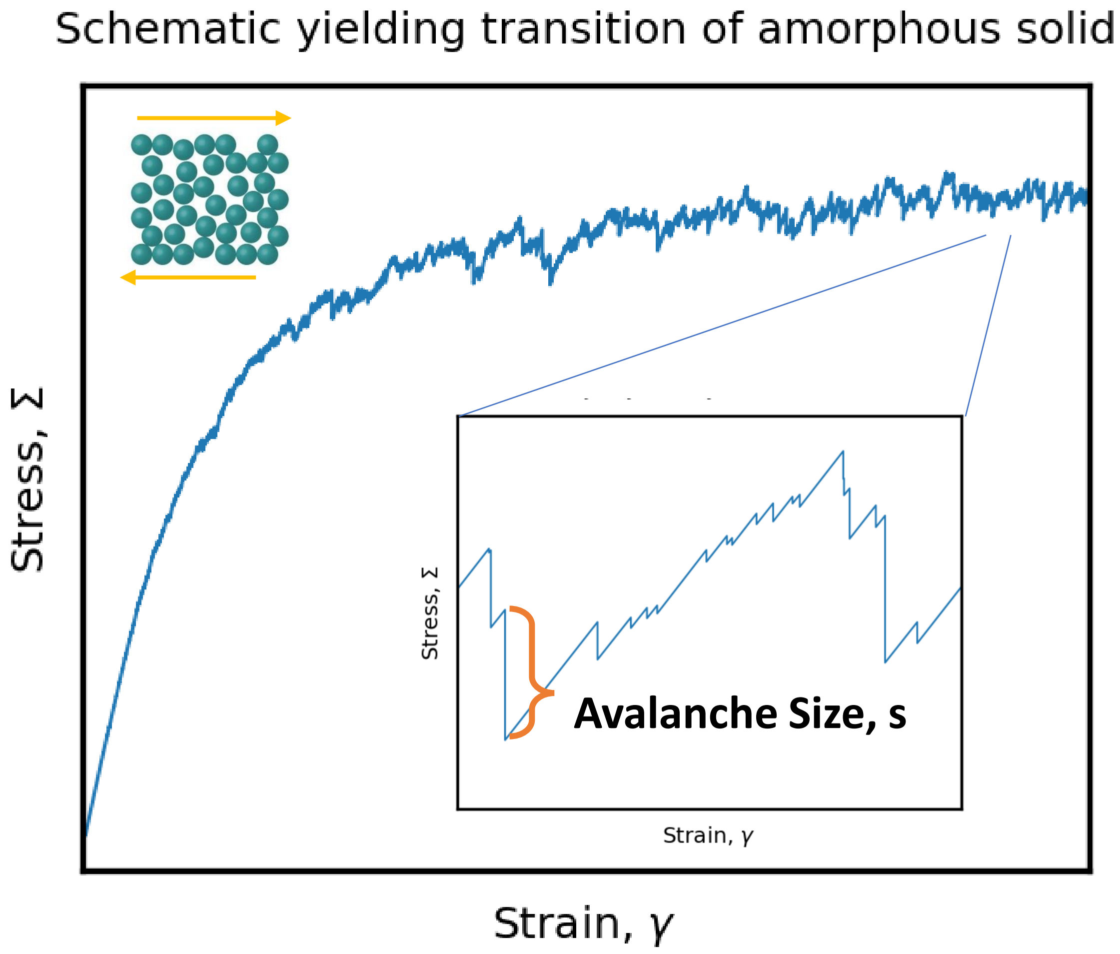

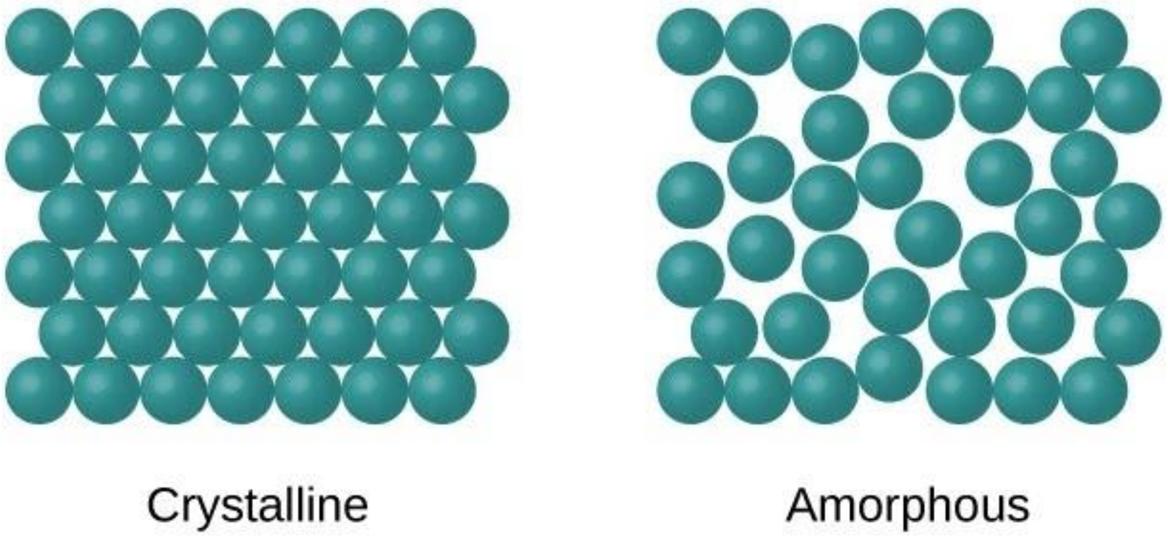

Amorphous solid graphic: opentextbc.ca

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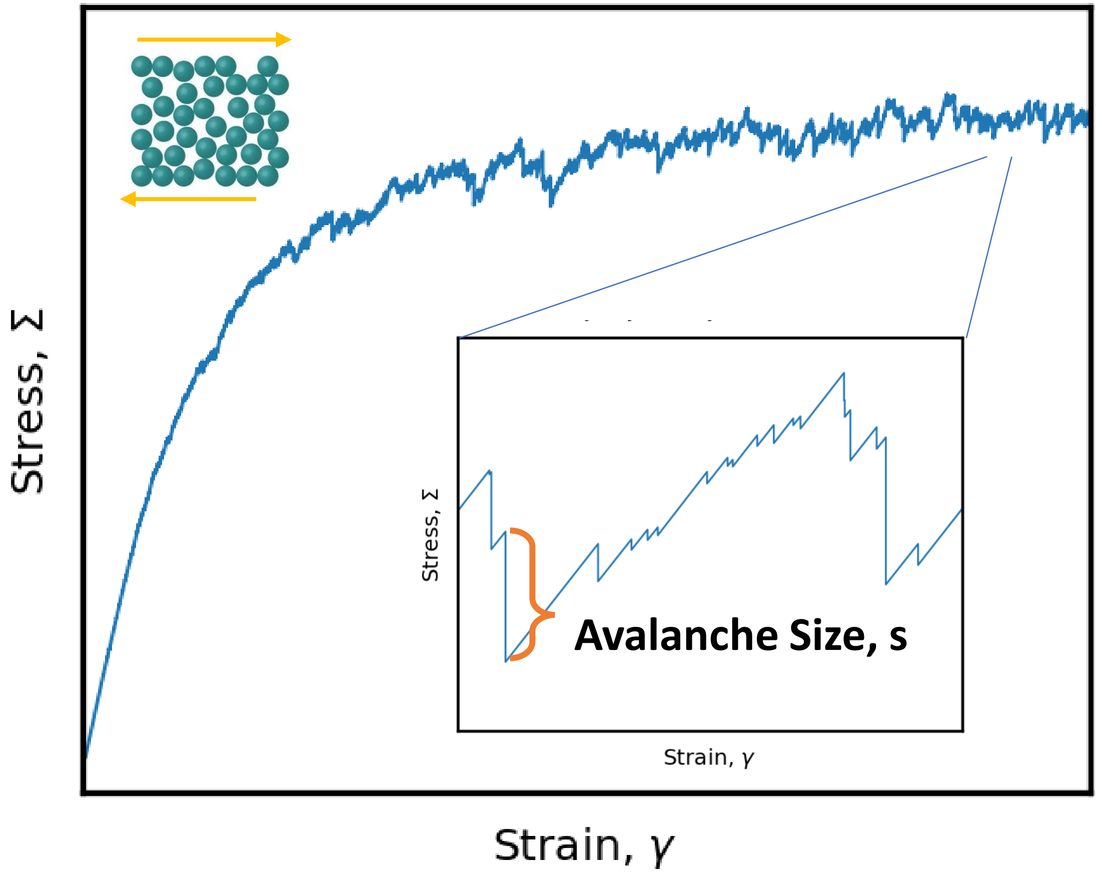
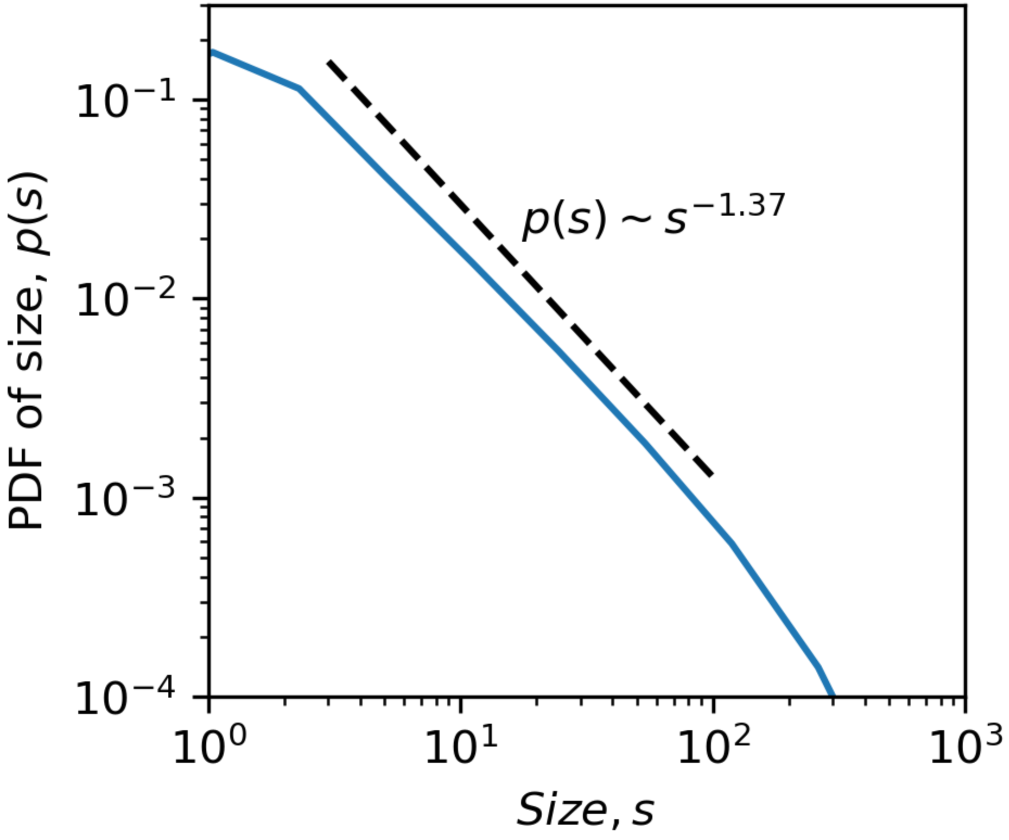
Amorphous yielding transition

Schematic yielding transition of amorphous solid



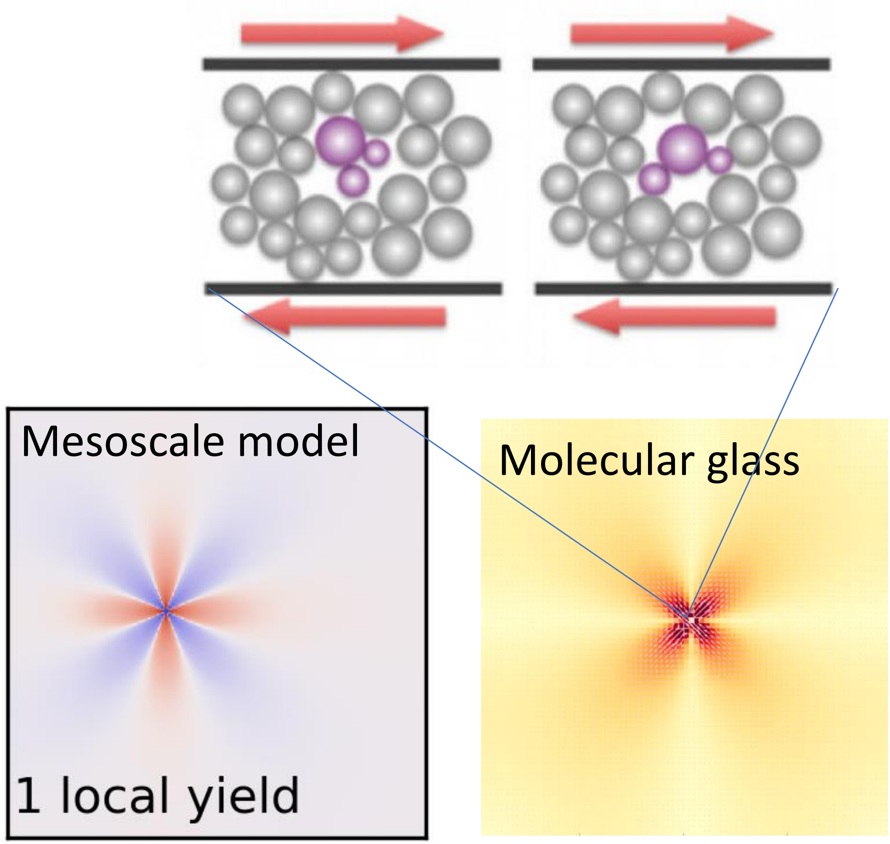
Universal scale-free avalanches



Amorphous solid graphic: opentextbc.ca

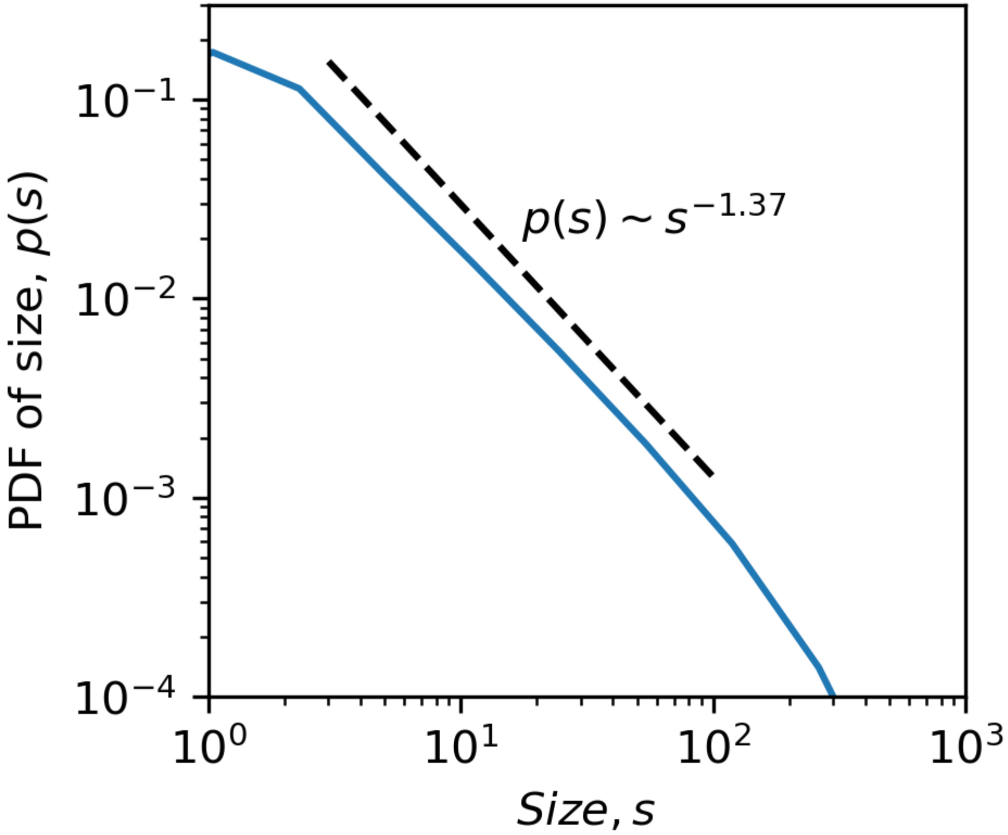
Amorphous yielding transition

- Common feature: Local shear transformations



(Eshelby) Stress Fields

Universal scale-free avalanches

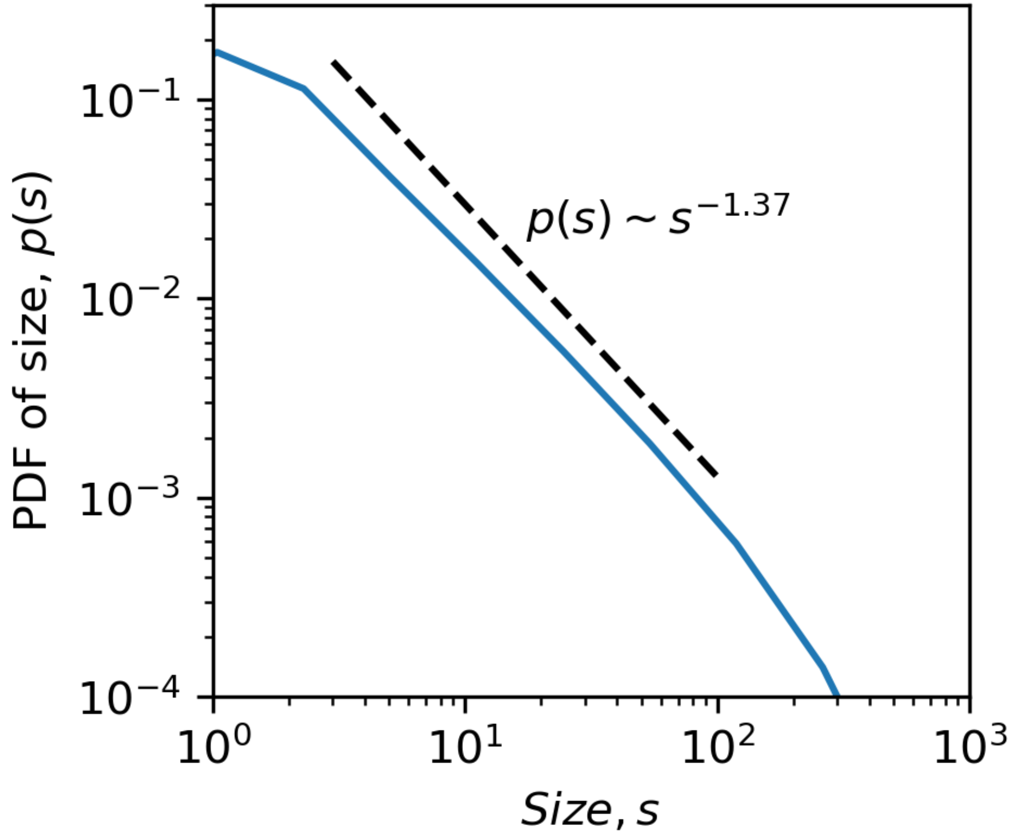


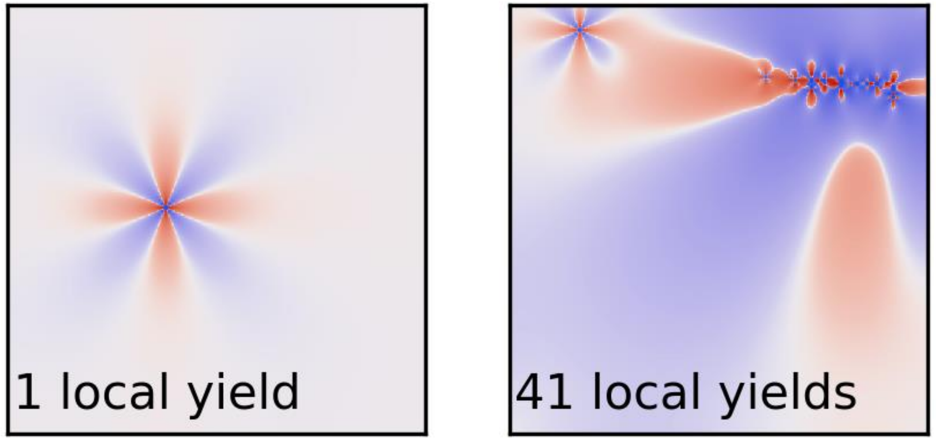
Amorphous solid graphic: opentextbc.ca

Amorphous yielding transition

- Common feature: Local shear transformations
- Beautiful scaling theory in athermal quasistatic (AQS) limit, distinct from depinning
- Avalanches proceed through shear-transformations with quadrupolar interactions

Universal scale-free avalanches





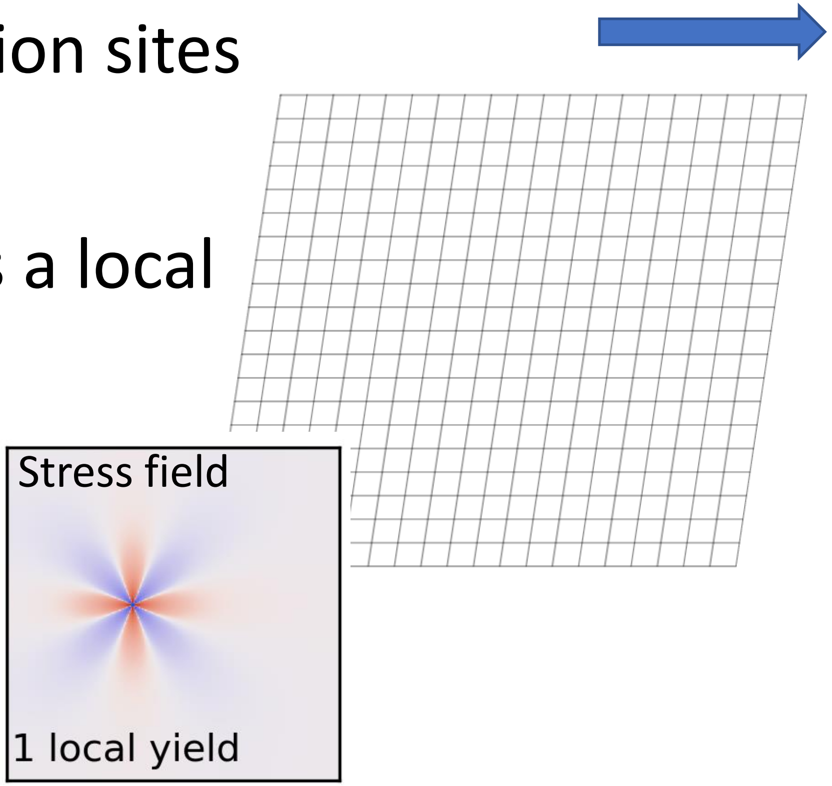
Stress Fields

Amorphous solid graphic: opentextbc.ca

Athermal Mesoscopic Model of Amorphous Yielding

Coarse grain to level of shear transformation sites

- Sites elastically coupled (finite element)
- Site i yields when local stress Σ_i exceeds a local threshold $\Sigma_{y,i}$, i.e. $x_i = \Sigma_{y,i} - |\Sigma_i| = 0$
- Site yield time $\tau_{plastic}$



For review of mesoscopic models, see Nicolas et al. Rev. Mod. Phys. **90**, 045006

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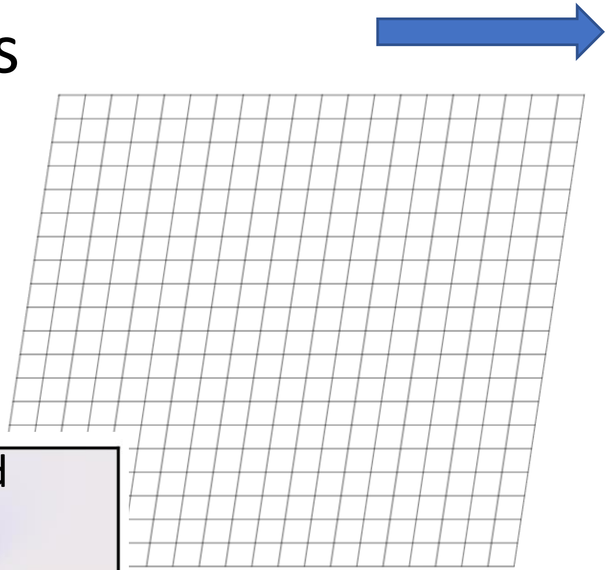
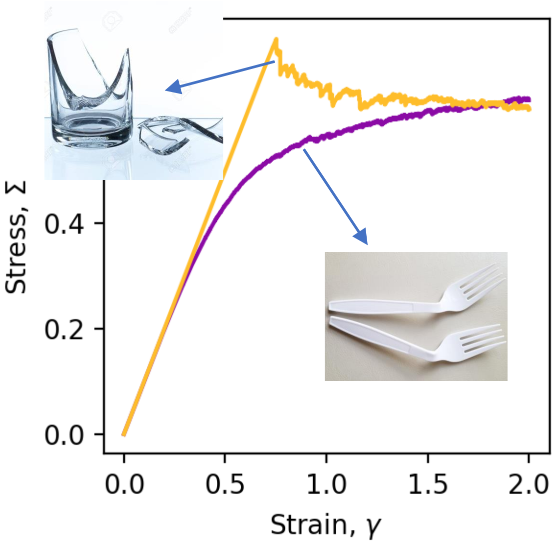
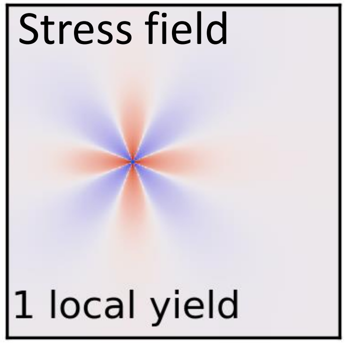


Brittle-Ductile transition is in initial annealed disorder





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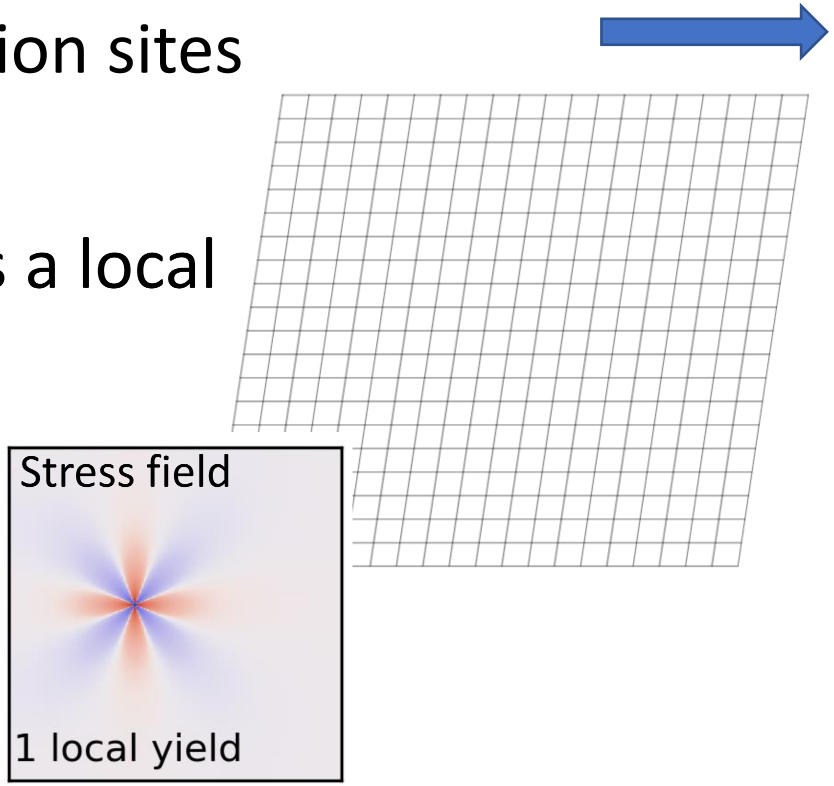
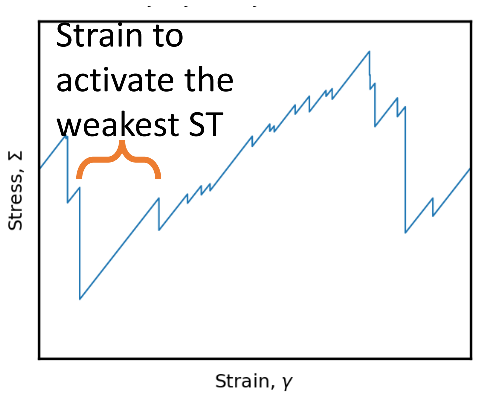
Residual stress





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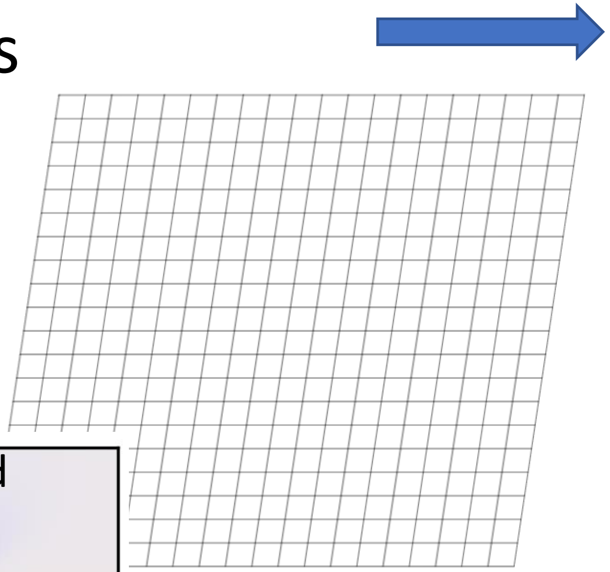

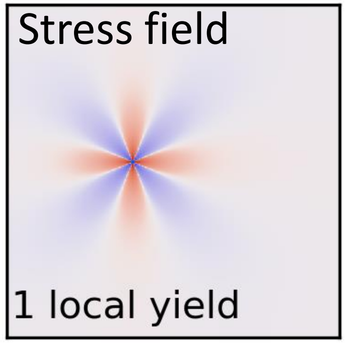

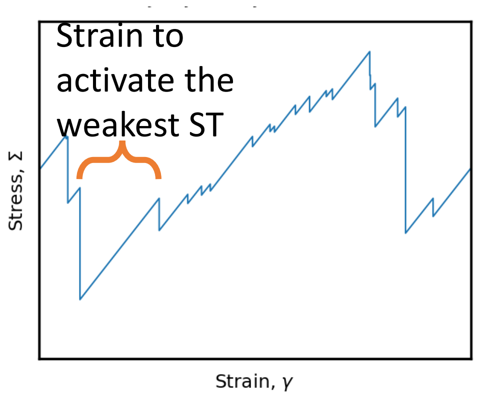

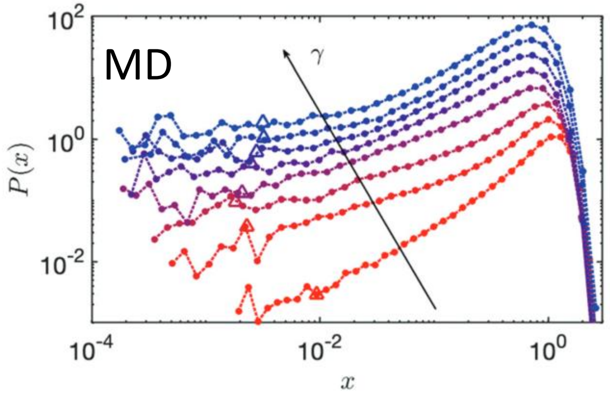

C. Ruscher and J. Rottler, Soft Matter 16, 8940 (2020).

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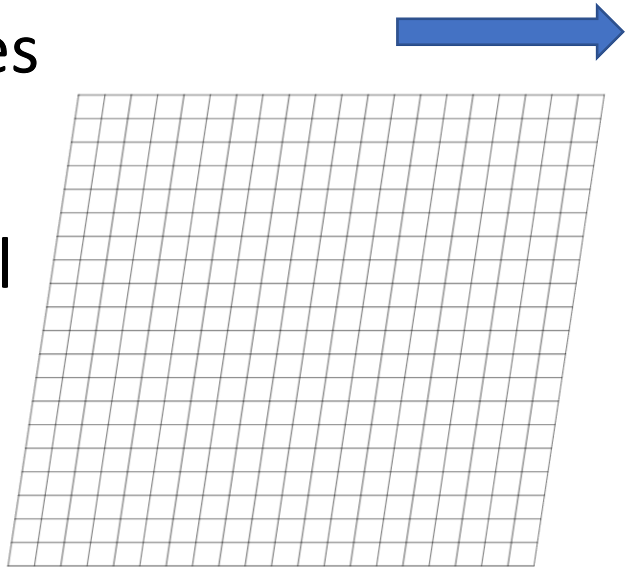

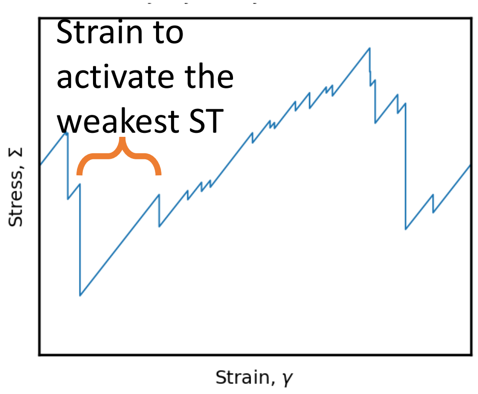

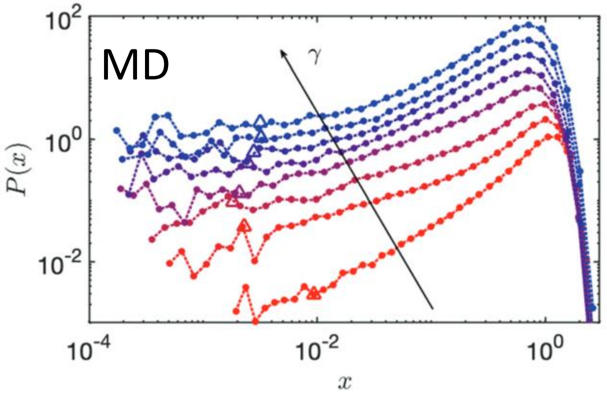

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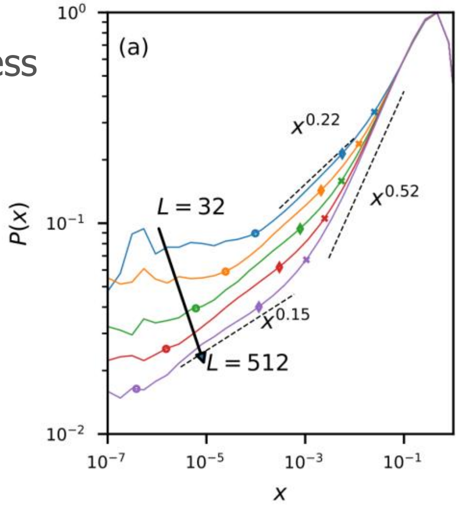


D. Korchinski, C. Ruscher, and J. Rottler, Phys. Rev. E **104**, 034603 (2021).

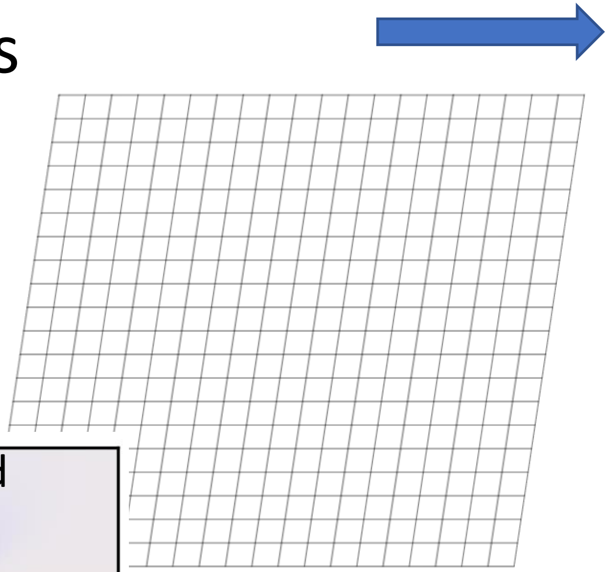
- Strong finite-size effects
- Interdependence with avalanche scaling

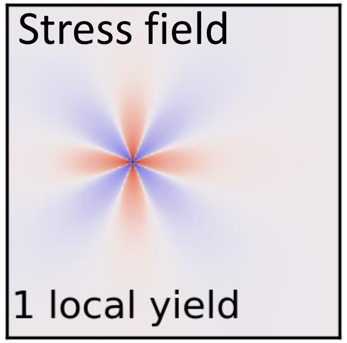
Thermal Mesoscopic Model of Amorphous Yielding

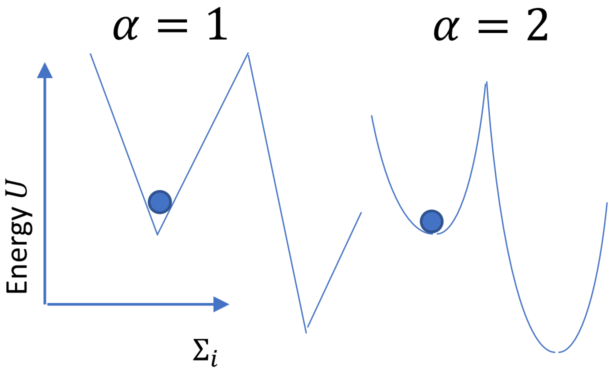
Coarse grain to level of shear transformation sites

- Sites elastically coupled (finite element)
- Site i yields when local stress Σ_i exceeds a local threshold $\Sigma_{y,i}$, i.e. $x_i = \Sigma_{y,i} - |\Sigma_i| = 0$
- Or stochastically, with Arrhenius rate

$$\lambda(x) = \frac{1}{\tau_{plastic}} \exp\left[-\frac{\mathrm{x}^{\alpha}}{\mathrm{T}}\right]$$





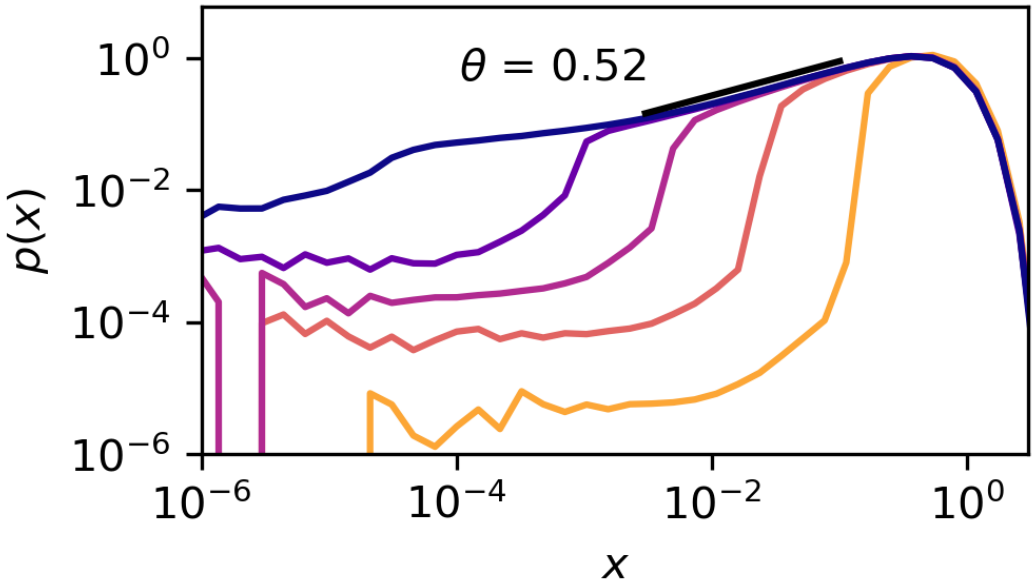


See:
Marko Popović et al. 2021
Ezequiel Ferrero et al. 2021
For studies of this model and rheological temperature dependence

For review of mesoscopic models, see Nicolas et al. Rev. Mod. Phys. **90**, 045006

Results: Residual stress distribution

- $p(x) \sim x^{\theta}$ for $T = 0$ and large L



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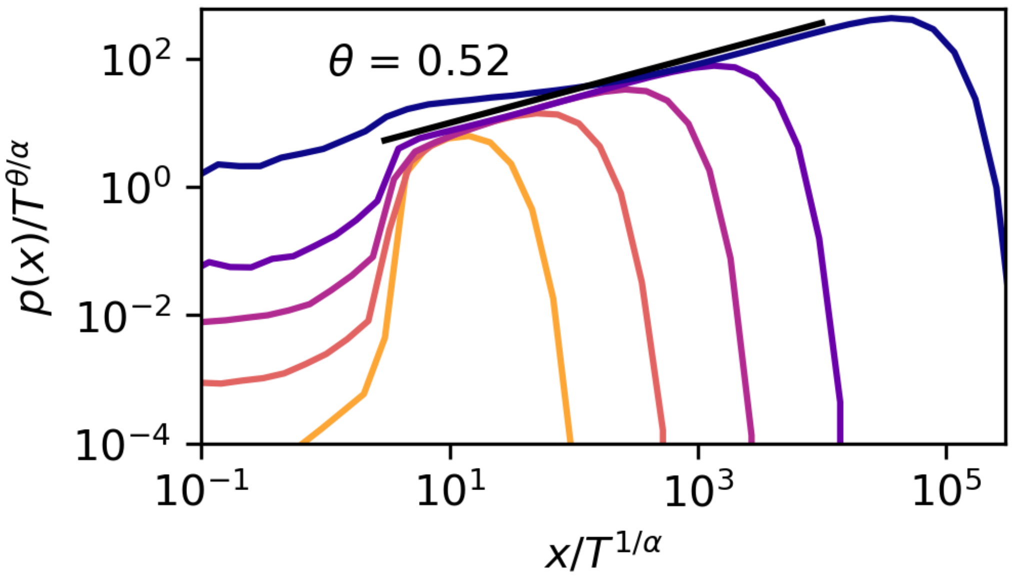
- Thermal activation scale: $x_c \sim T^{\frac{1}{\alpha}}$



What happens to avalanches with temperature?

- Partly answered in molecular dynamics (See: Karmakar et al. *PRE.* 2010)
  - Expect driving rate / temperature dominated regimes
  - Crossovers depend on system size
  - Herschel-Bulkley stress-rise occurs as avalanches overlap

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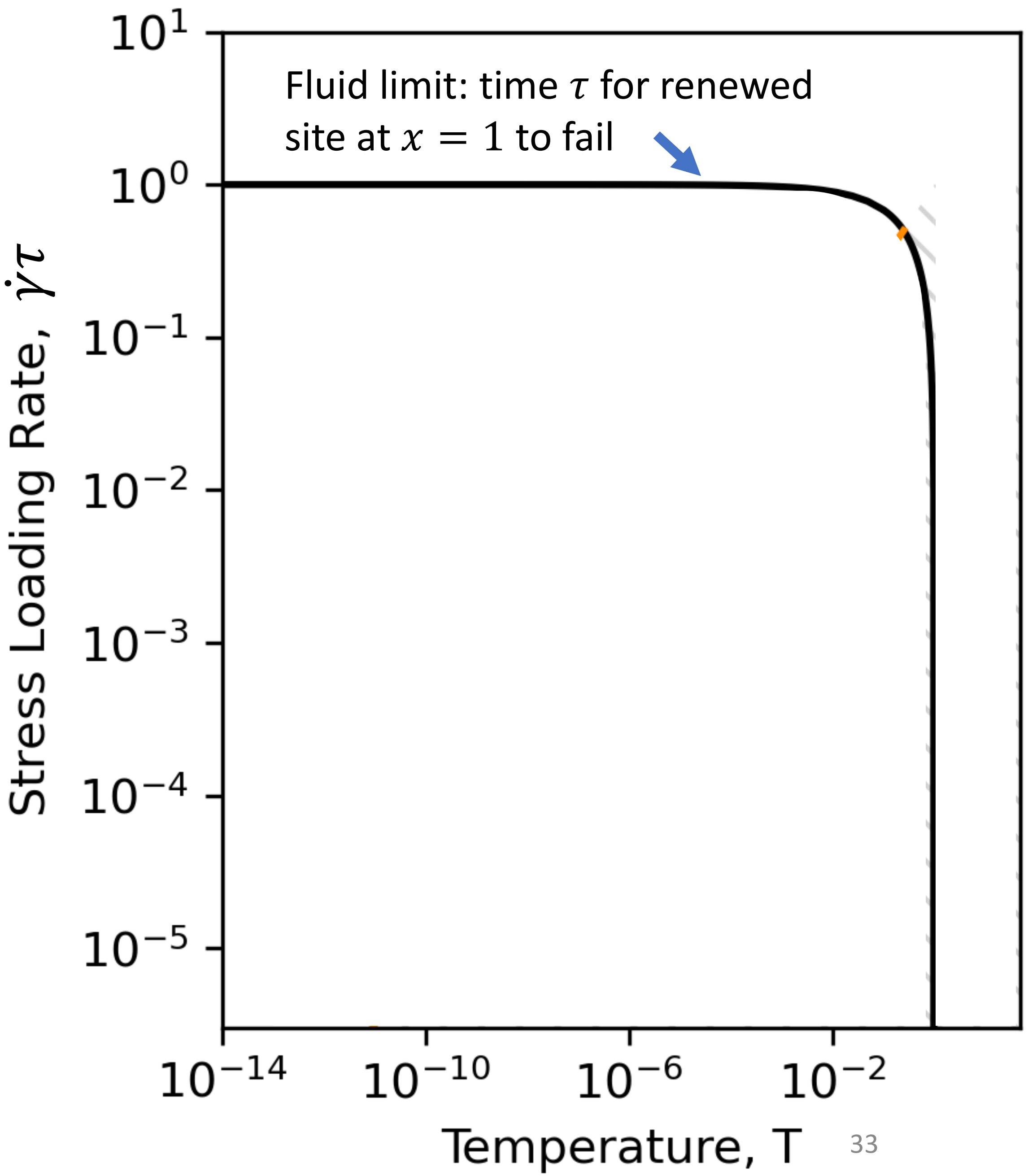
- Partly answered in molecular dynamics (See: Karmakar et al. *PRE.* 2010)
  - Expect driving rate / temperature dominated regimes
  - Crossovers depend on system size
  - Herschel-Bulkley stress-rise occurs as avalanches overlap
- Elastoplastic models expose several new aspects:
  - Residual stress distribution
  - Can probe very long timescales / low temperatures

Results: Phase diagram

- Most phase lines originate from competition of timescales
- Main timescales
  - $\tau_{plastic}$ the ST plastic time

Phase Diagram of Thermal EPM



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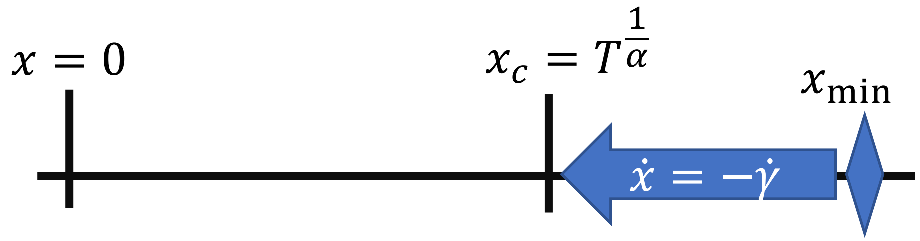

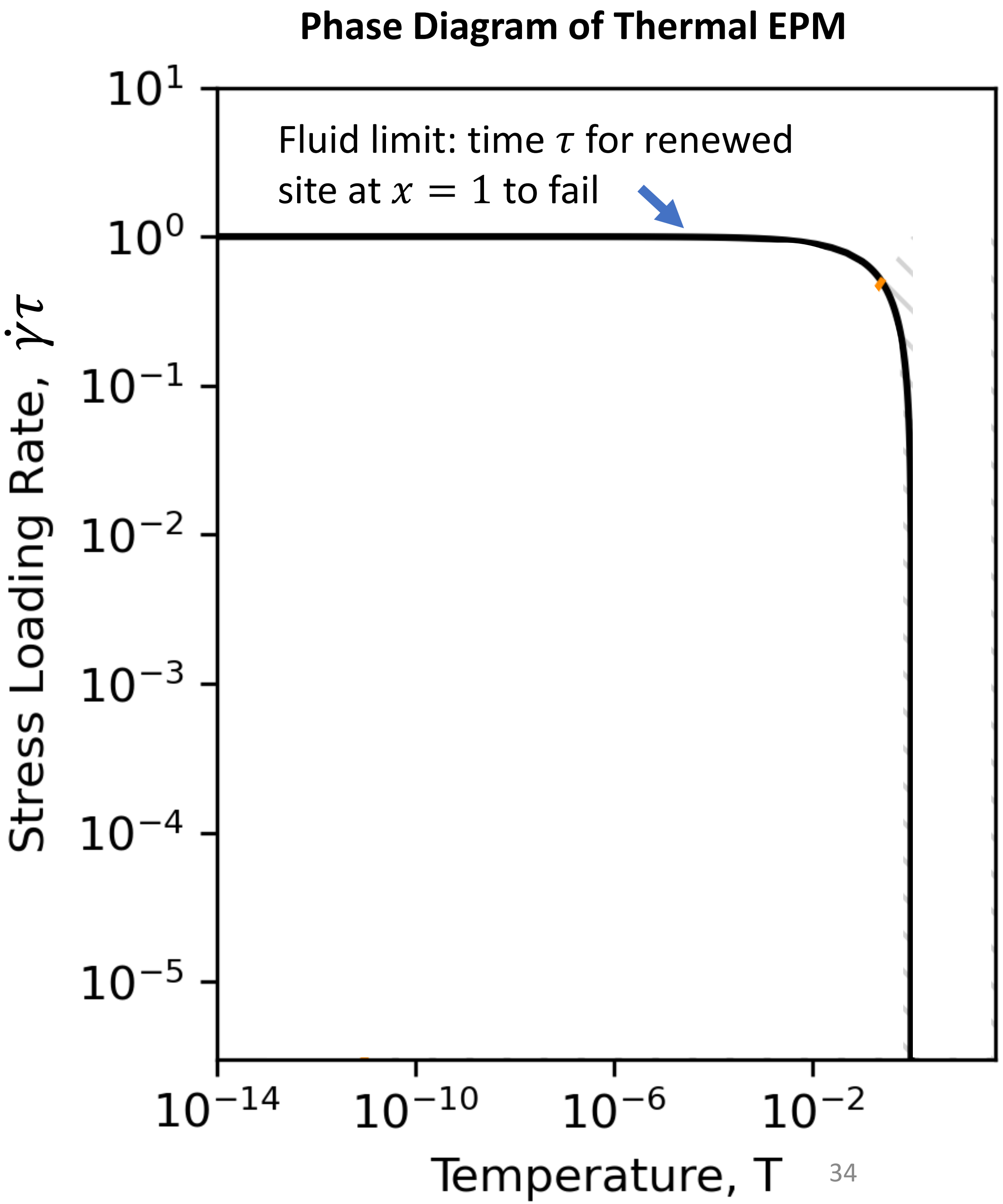

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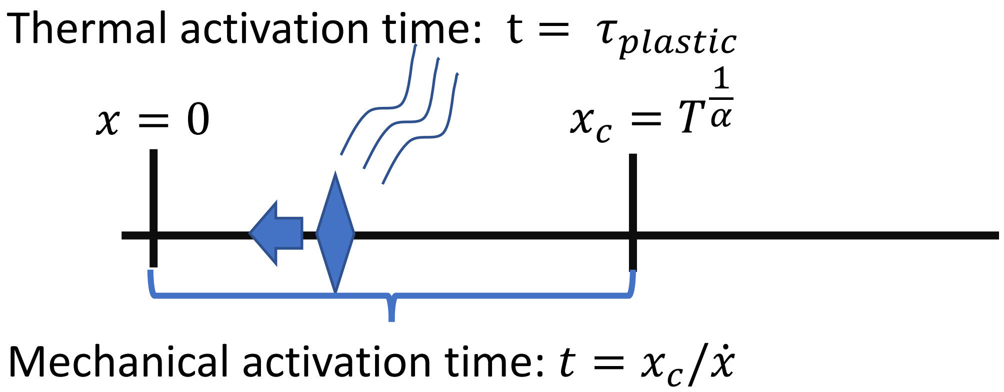

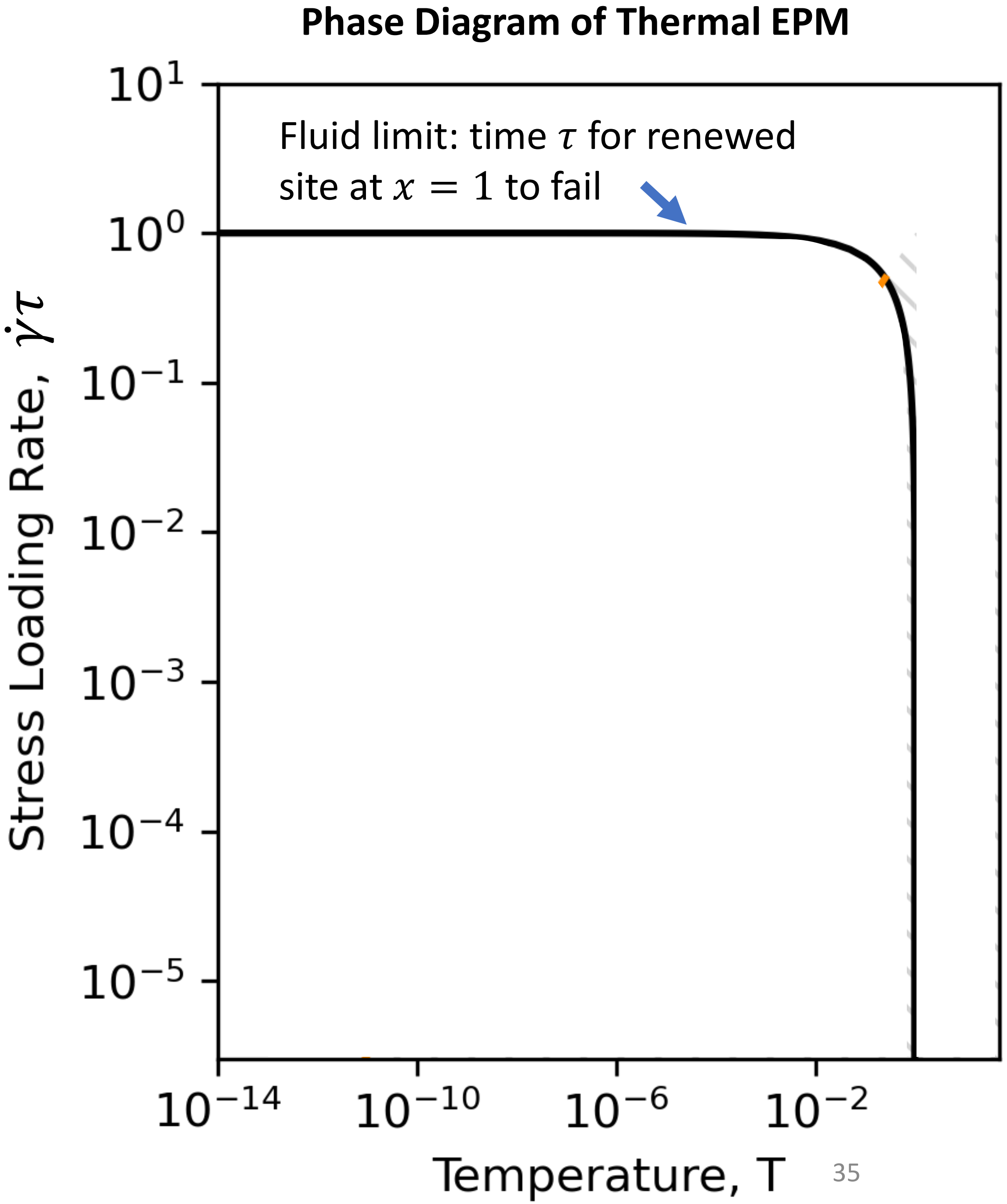

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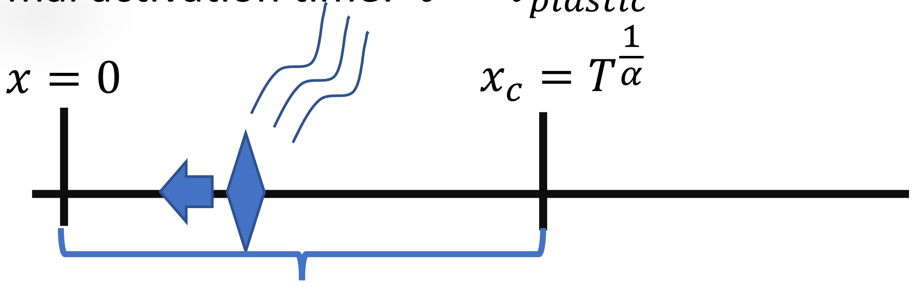
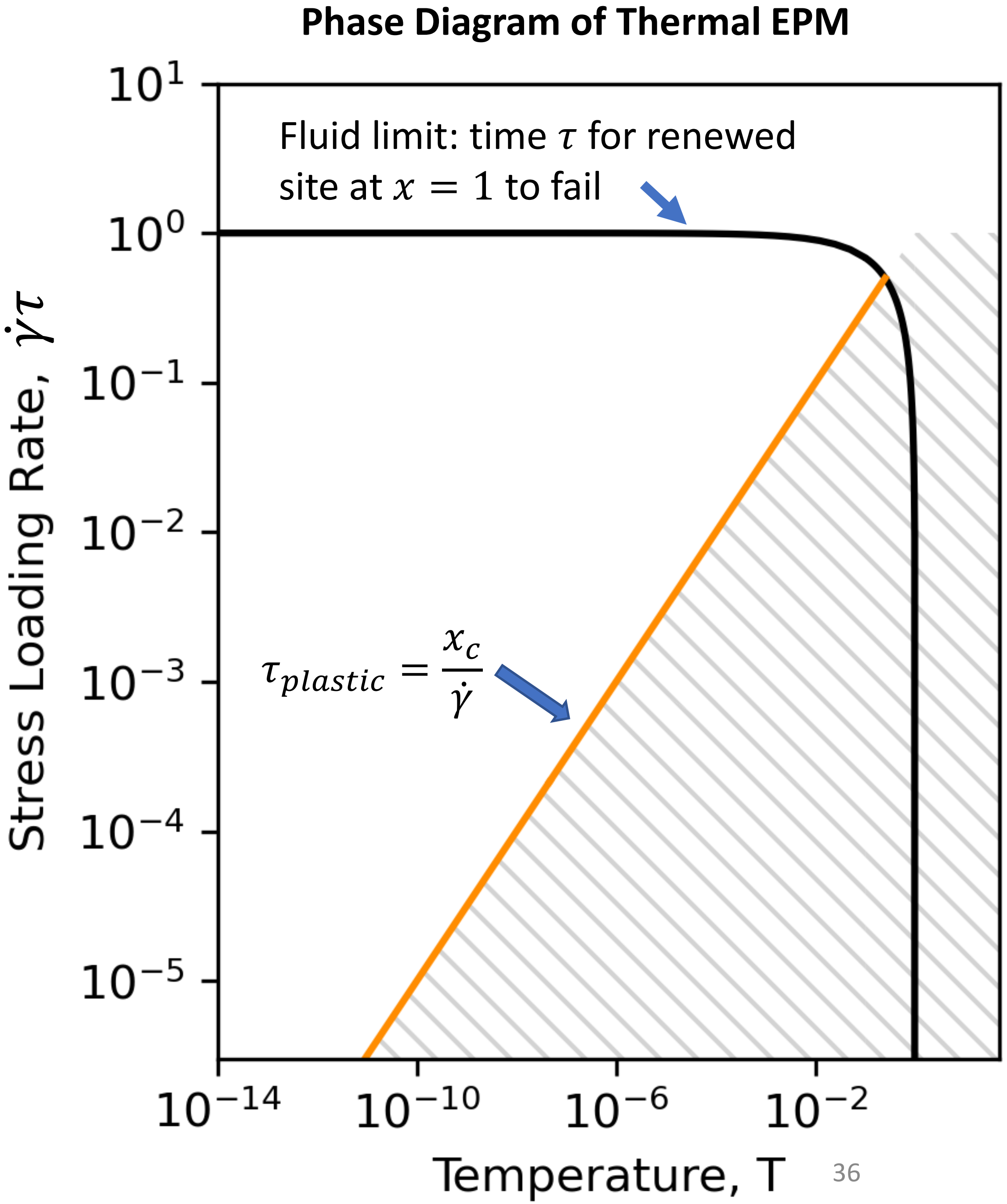
Temperature effects $\gg$ driving rate effects

Thermal activation time: $t = \tau_{plastic}$



Mechanical activation time: $t = x_c/\dot{x}$

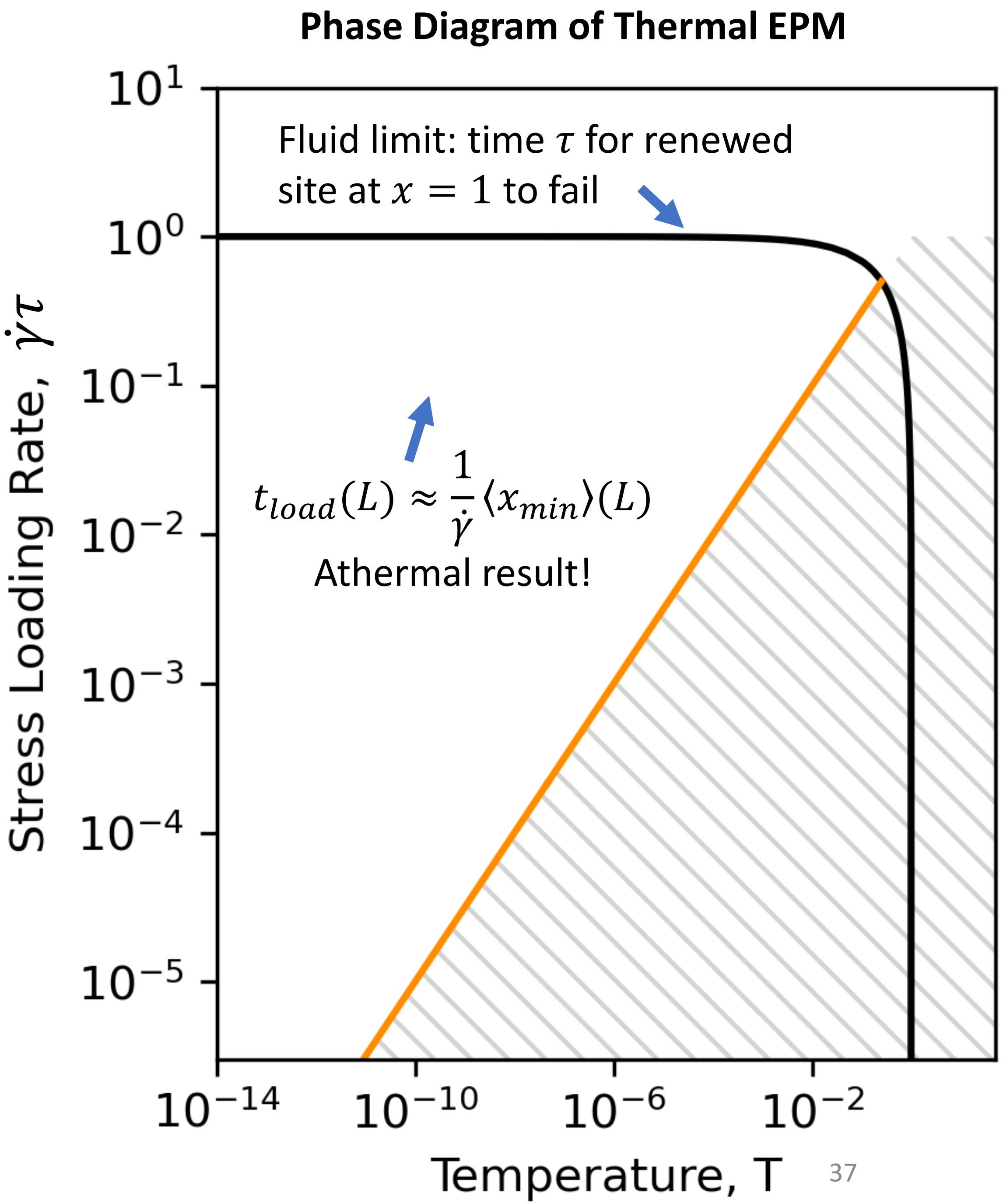


Results: Phase diagram

- Most phase lines originate from competition of timescales
- Main timescales
  - $\tau_{plastic}$ the ST plastic time
  - t_{load} between avalanches

Calculated using extreme value statistics on $p(x)$

Temperature effects ≫ driving rate effects



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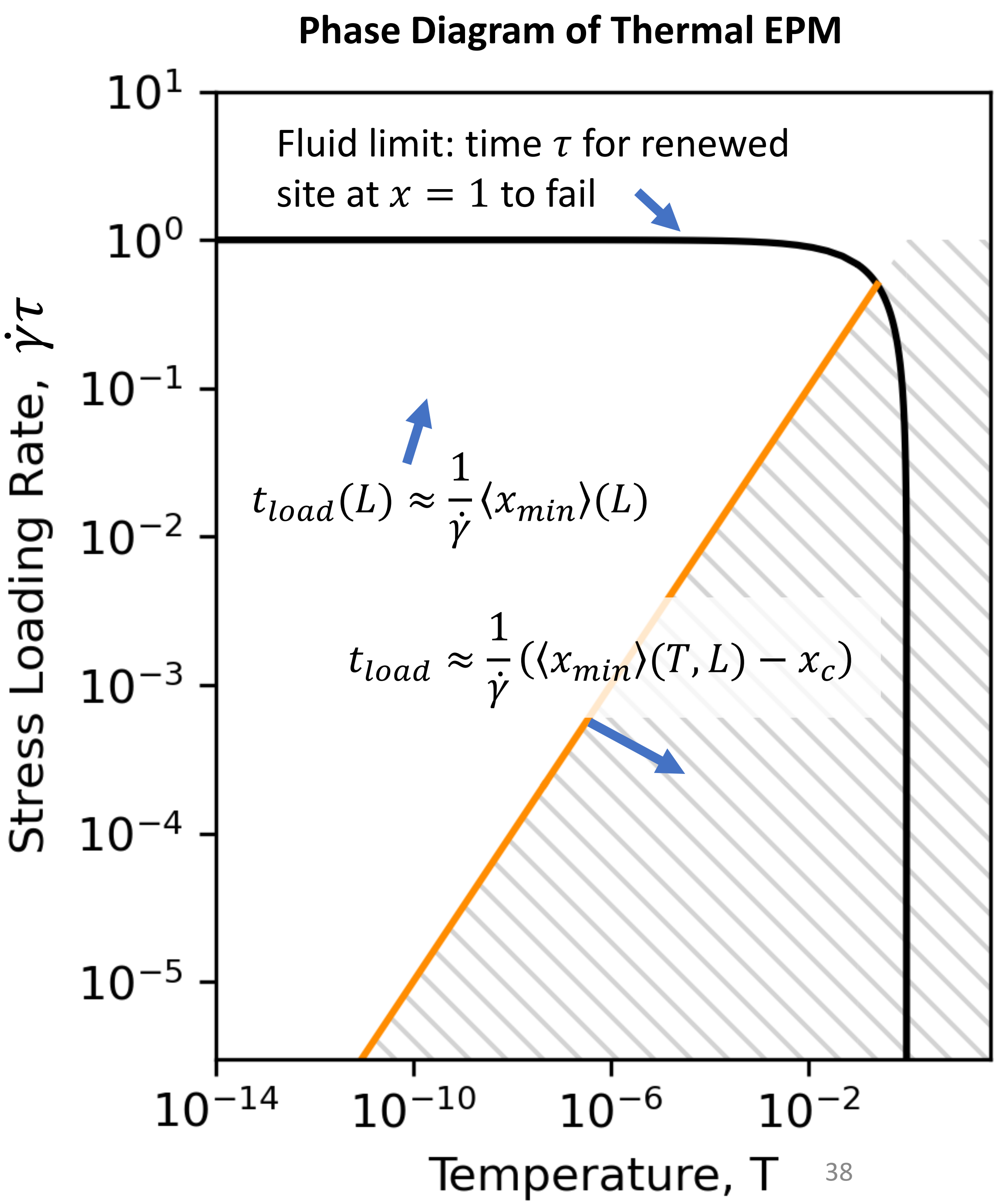
Temperature effects ≫ driving rate effects

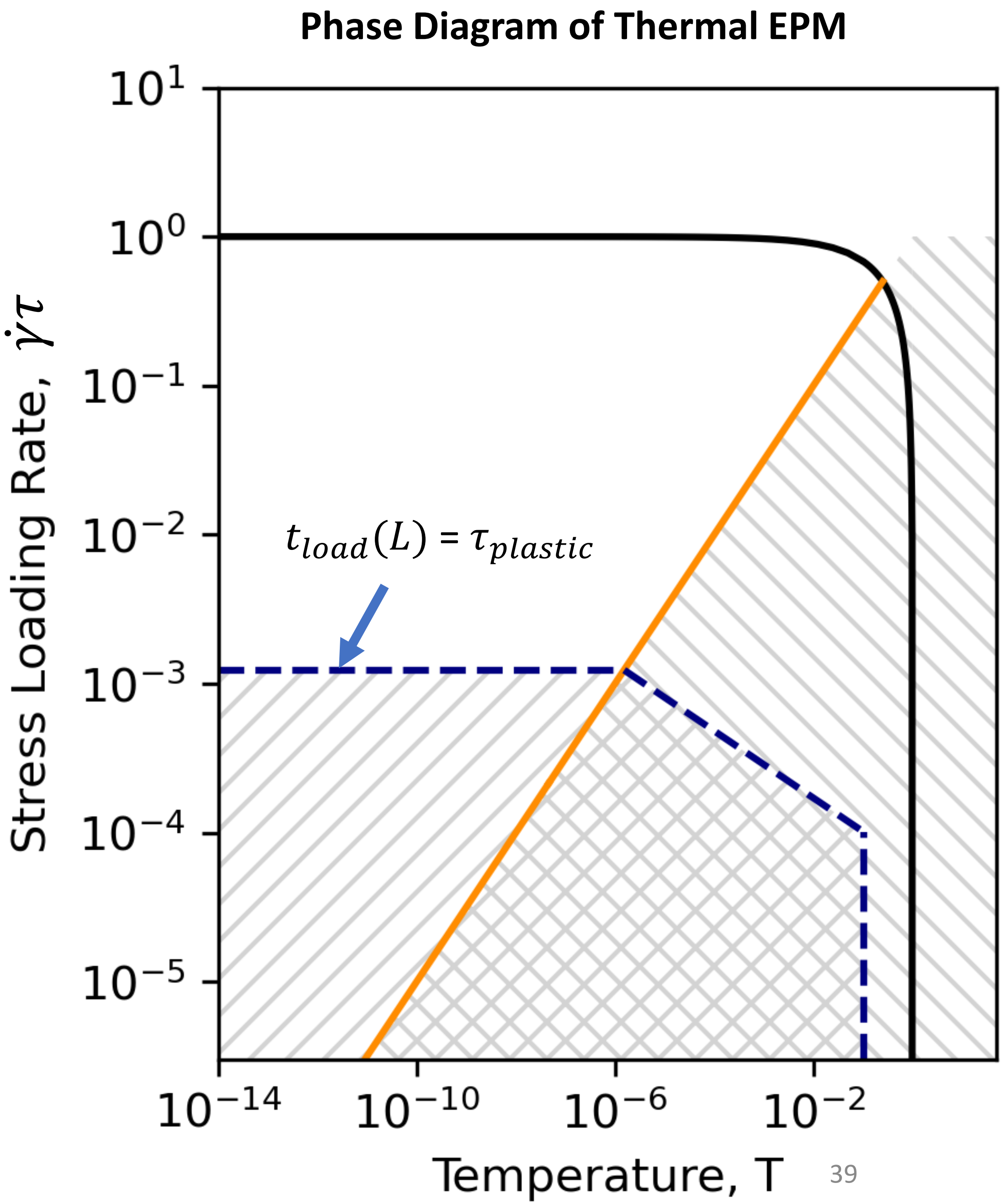


Results: Phase diagram

- Most phase lines originate from competition of timescales
- Main timescales
  - $\tau_{plastic}$ the ST plastic time
  - t_{load} between avalanches

Temperature effects ≫ driving rate effects

Temporally distinct avalanches (L dependent)



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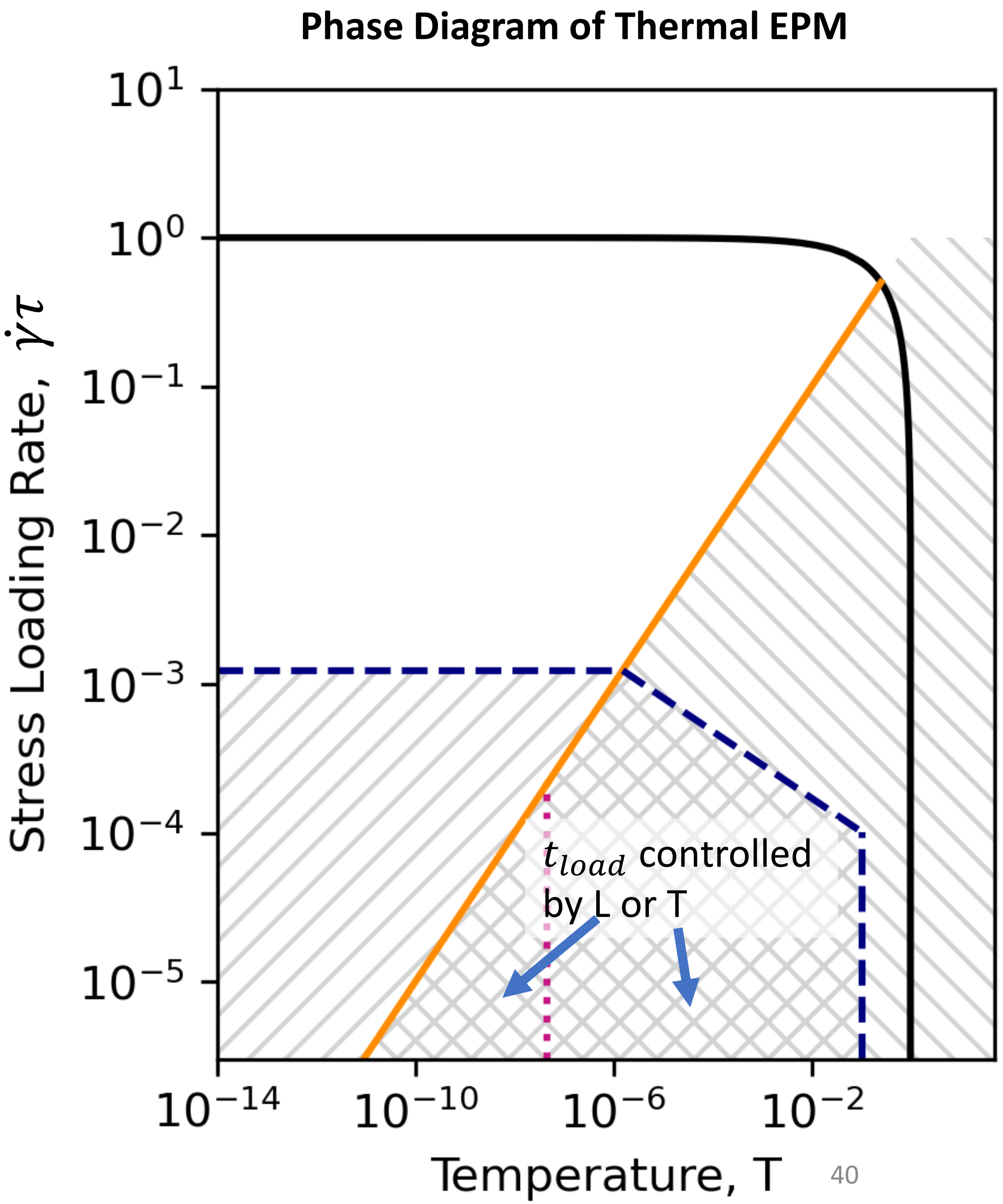

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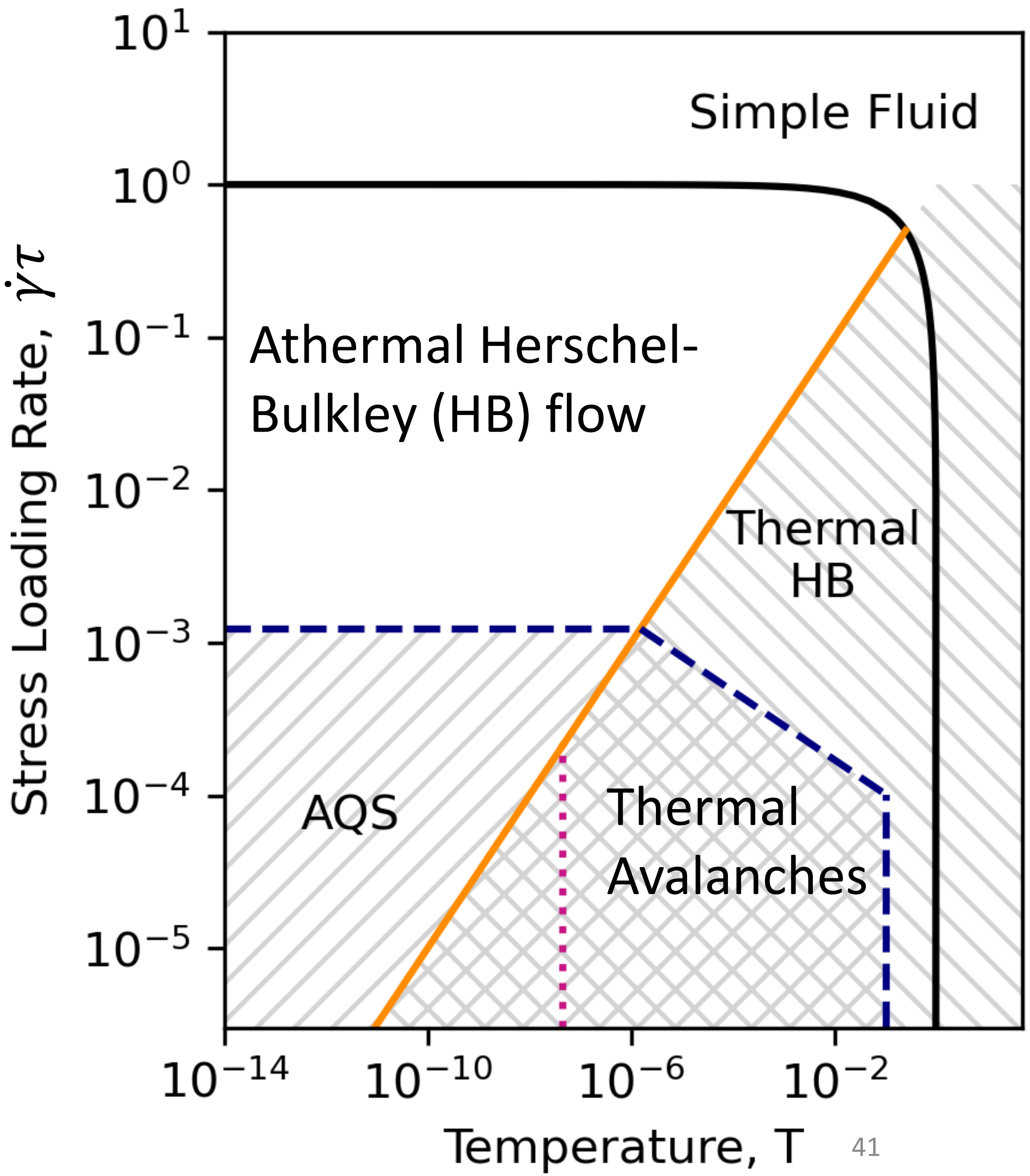
Phase Diagram of Thermal EPM



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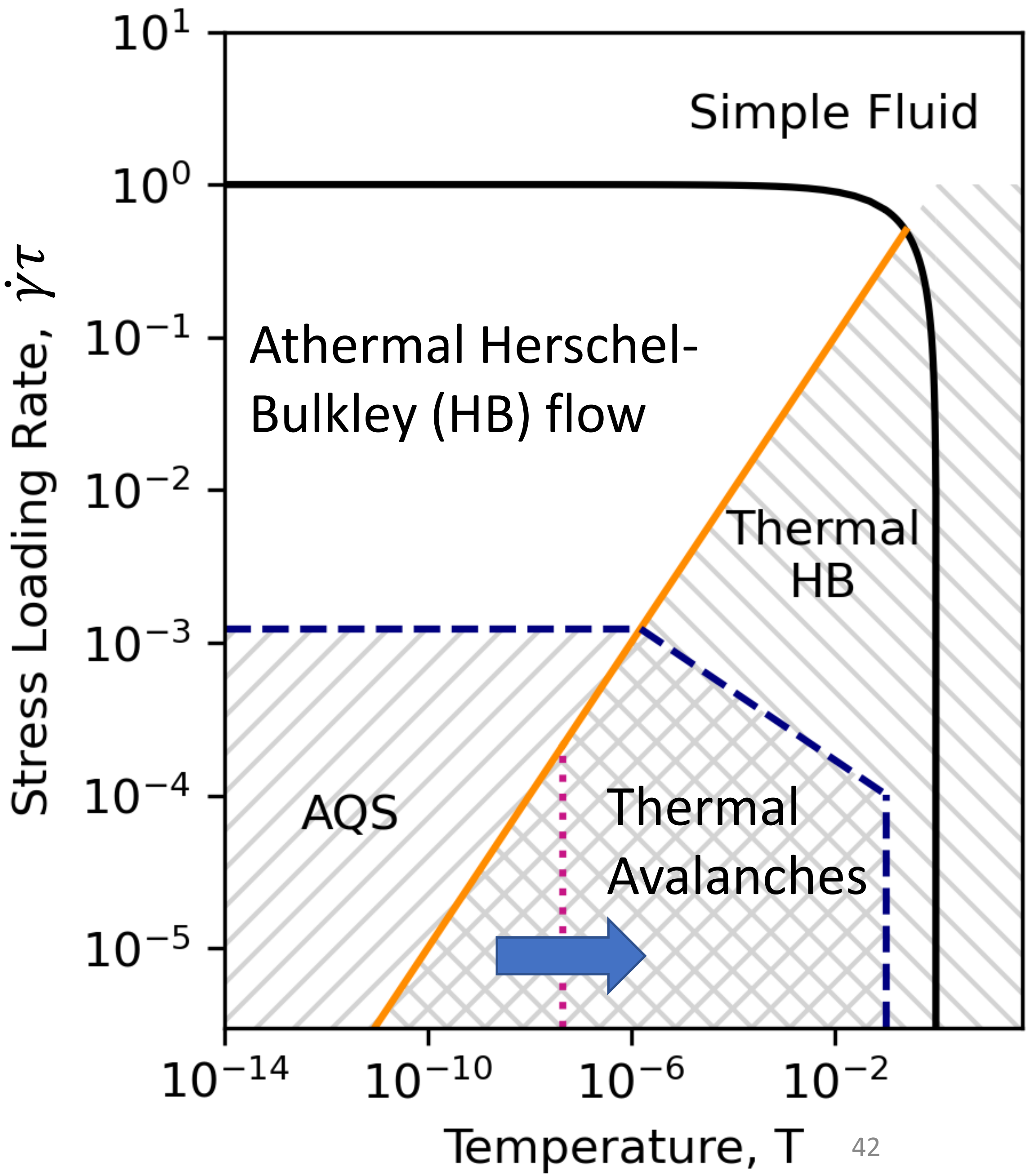


Results: Temperature truncated avalanches

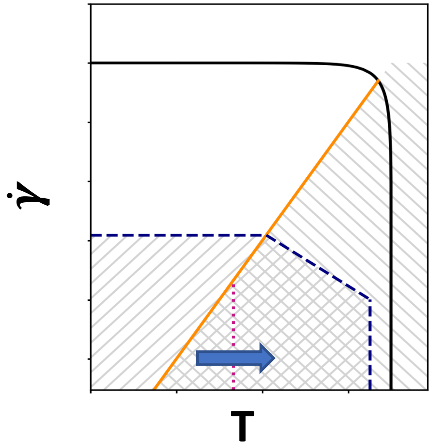
- Temperature reduces avalanche size:

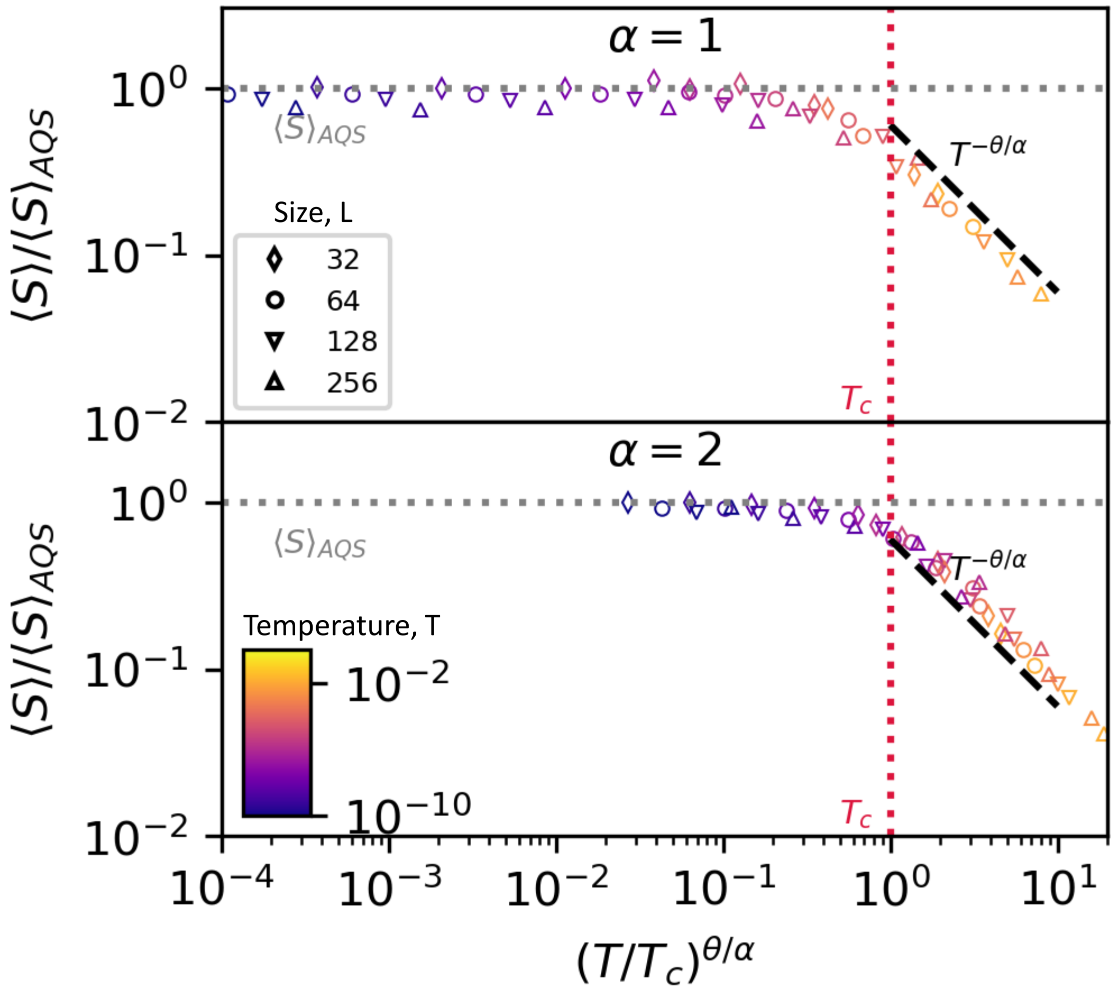
$$\langle S \rangle = t_{load}\,\dot{\gamma} \text{ (steady state)}$$

$$\langle S \rangle \sim T^{-\frac{\theta}{\alpha}} \quad \text{for} \quad T > T_c \sim L^{-\frac{d\alpha}{\theta+1}}$$

$$\langle S \rangle \sim L^{-\frac{d}{1+\theta}} \quad \text{for} \quad T < T_c$$

Crossing L, T phase line





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- Temperature reduces avalanche size when:

$$T > T_c \sim L^{-\frac{d\alpha}{\theta+1}}$$

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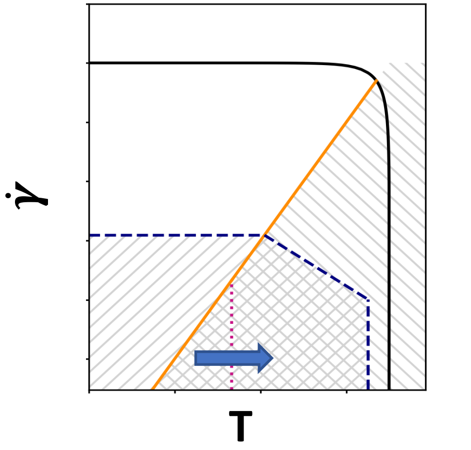
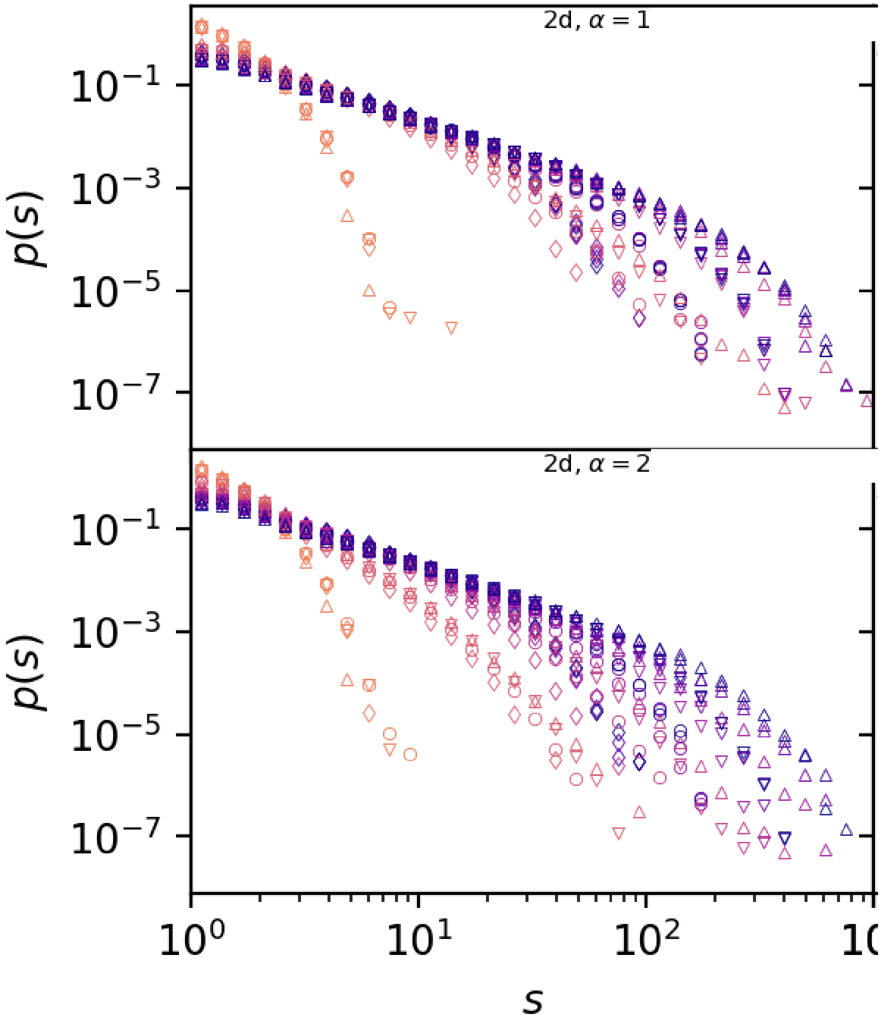




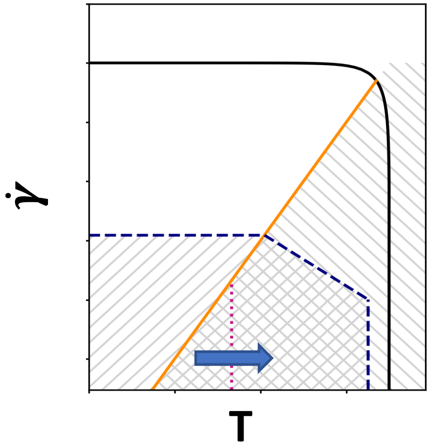
Results: Temperature truncated avalanches

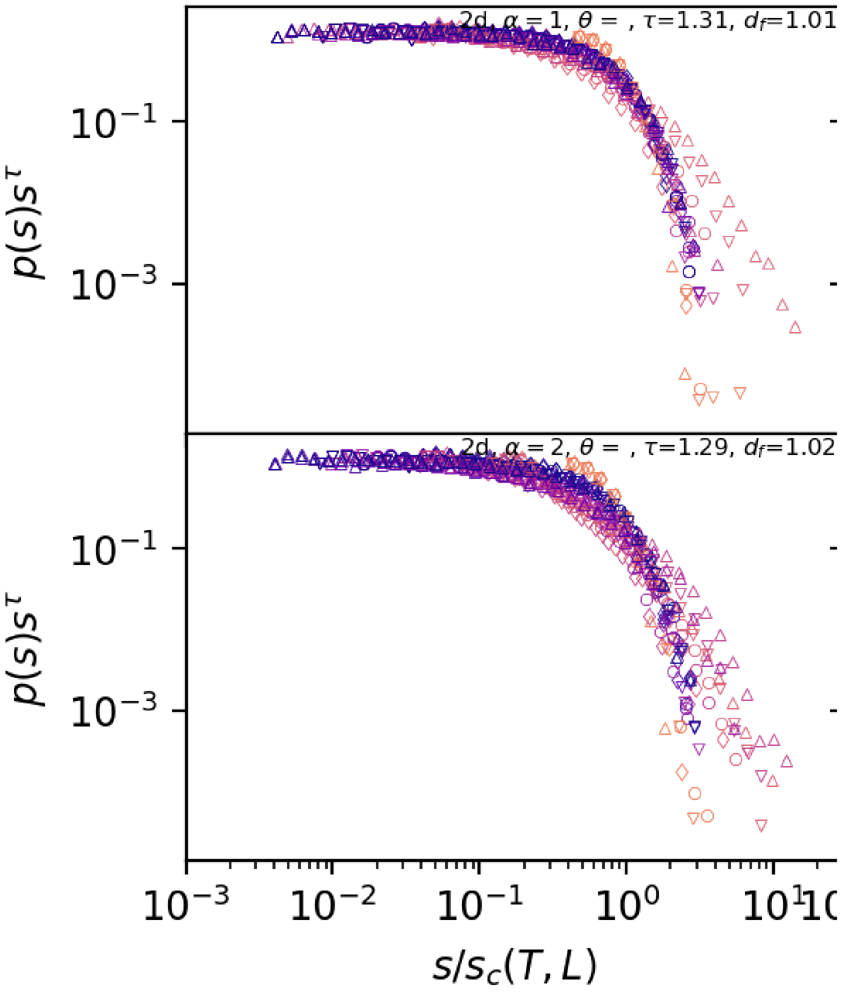
- Temperature reduces avalanche size when:

$$T > T_c \sim L^{-\frac{d\alpha}{\theta+1}}$$

- $s_c \sim \min(L, \xi(T))^{d_f}$
- Rescaling also works for avalanche duration

Crossing L, T phase line





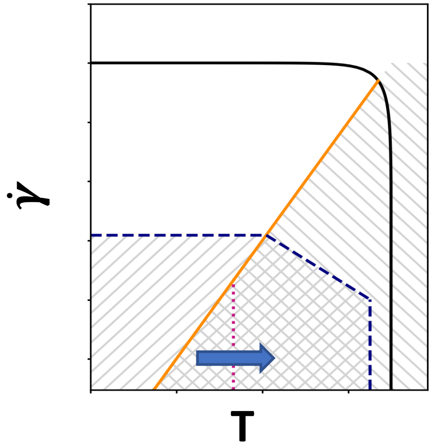
Results: Temperature truncated avalanches

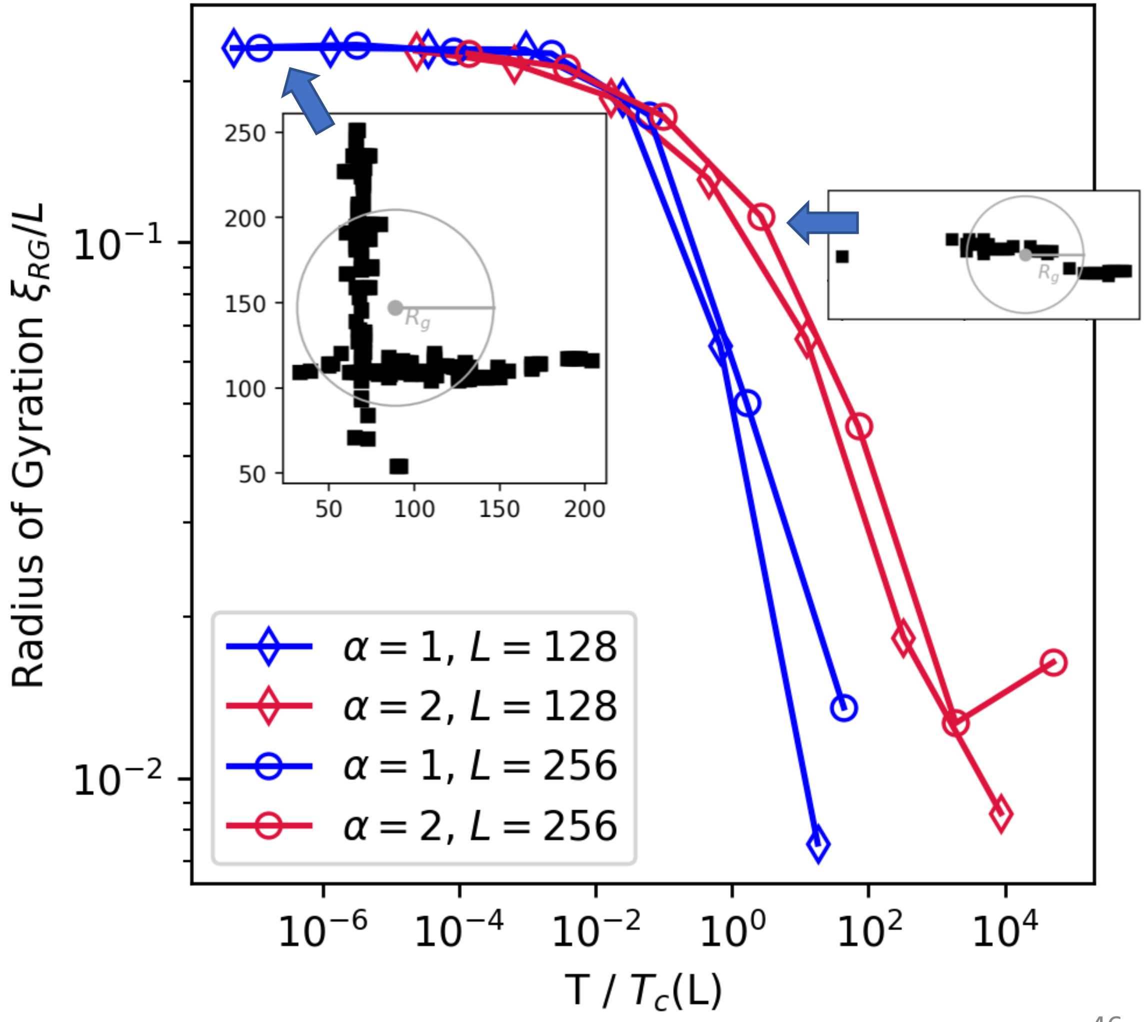
- Temperature reduces avalanche size when:

$$T > T_c \sim L^{-\frac{d\alpha}{\theta+1}}$$

- Interpretation: correlation length & avalanches truncated by either system size or temperature effects

Crossing L, T phase line

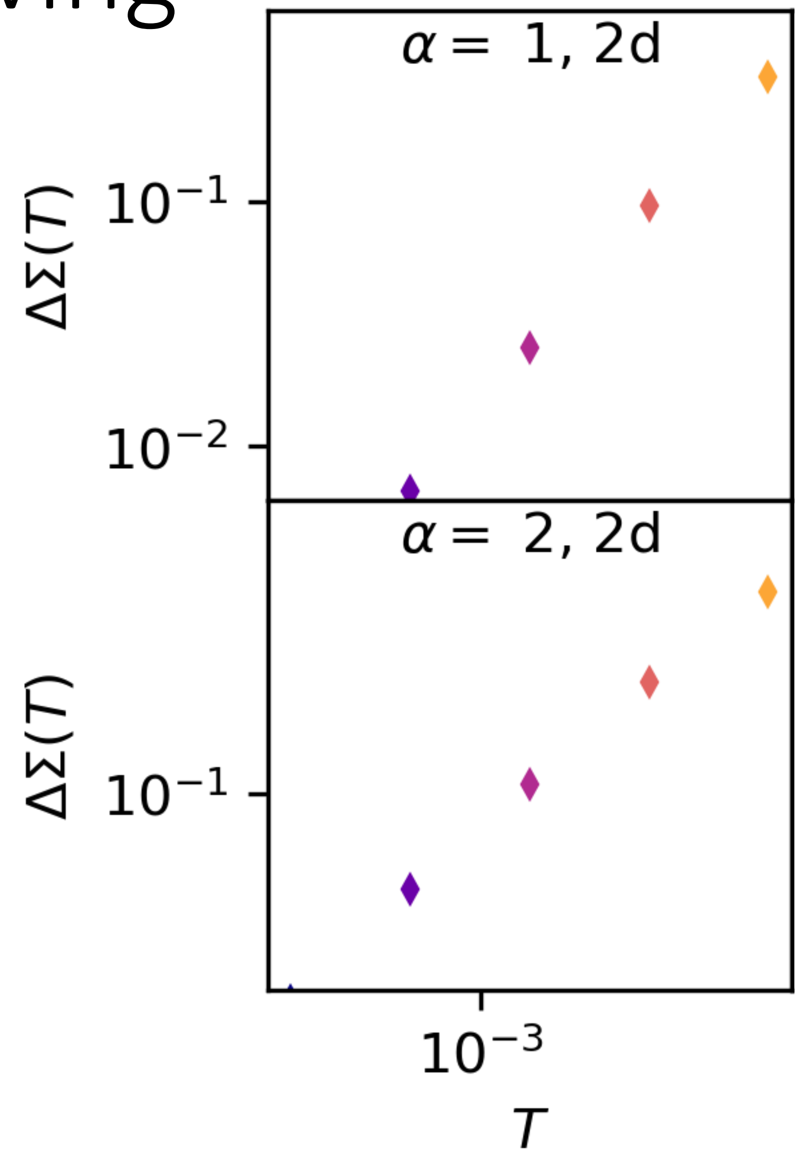




Material softening at low driving

- At low driving rates, system fluctuates around a lower flow stress that depends on temperature:

$$\langle\Sigma\rangle(T) = \Sigma_c - \Delta\Sigma(T)$$



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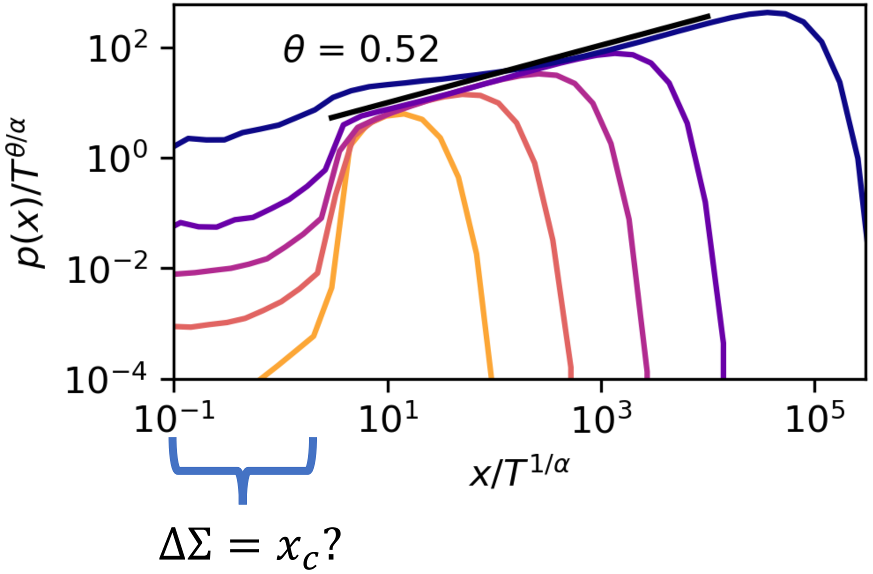

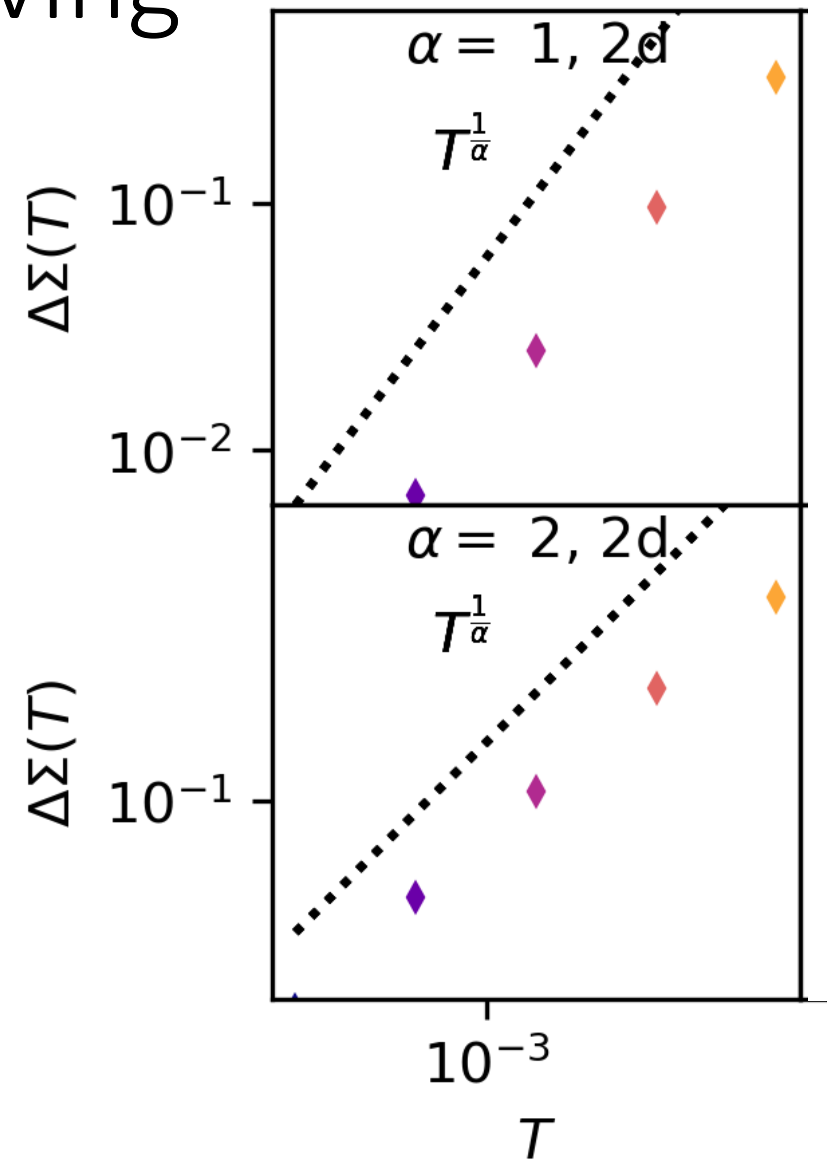

Material softening at low driving

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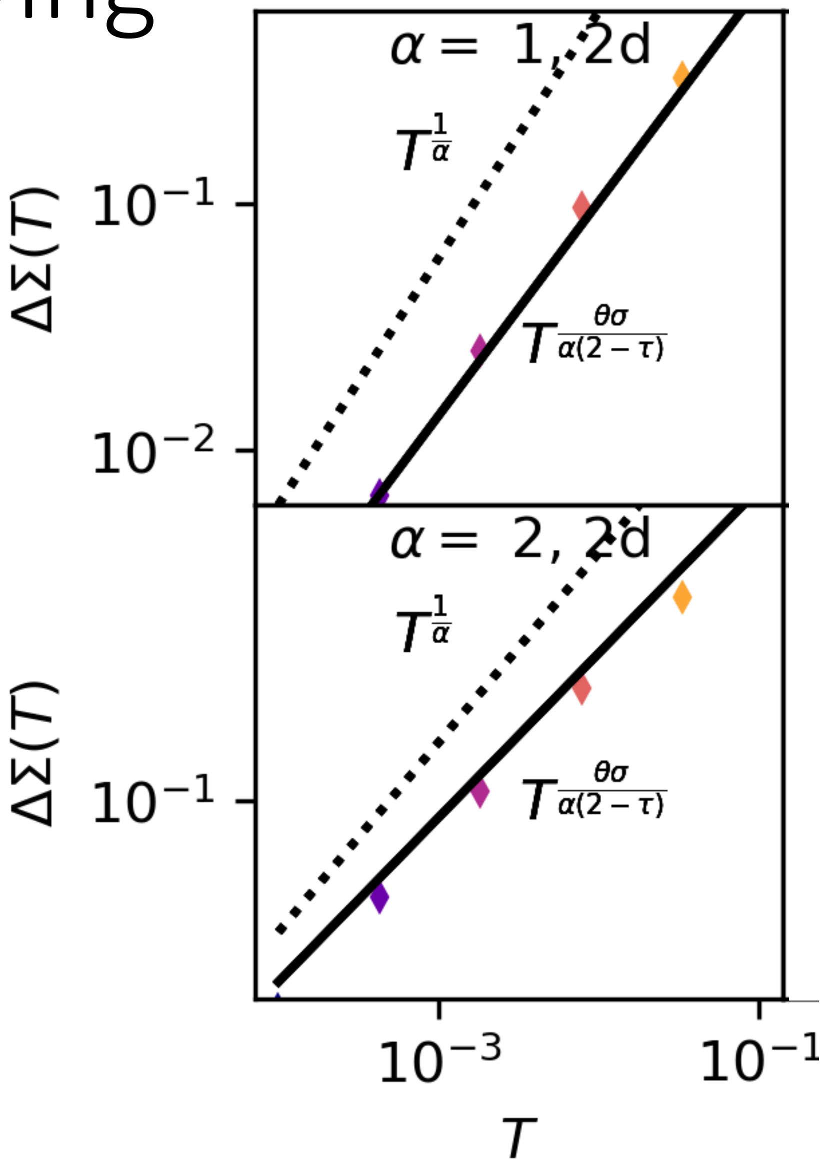
$$\langle\Sigma\rangle(T) = \Sigma_c - \Delta\Sigma(T)$$

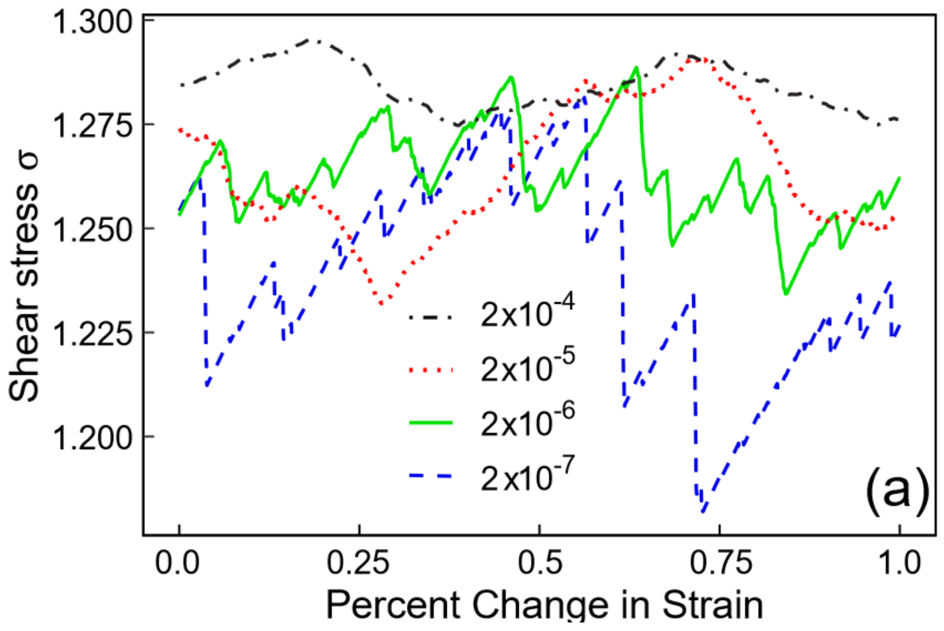
- Is the stress gap $\Delta\Sigma \sim x_c \sim T^{\frac{1}{\alpha}}$? No!

$$\Delta\Sigma \sim T^{\frac{\theta\sigma}{\alpha(2-\tau)}}$$

- Avalanches drive the stress gap



Results: Athermal rheology



Joel T. Clemmer, K. Michael Salerno, and Mark O. Robbins Phys. Rev. E 2021

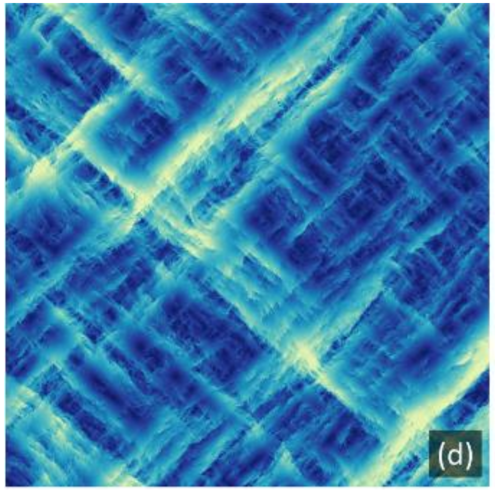
What happens under strong driving?

Avalanches overlap, stress rises ⇒ Herschel-Bulkley flow:

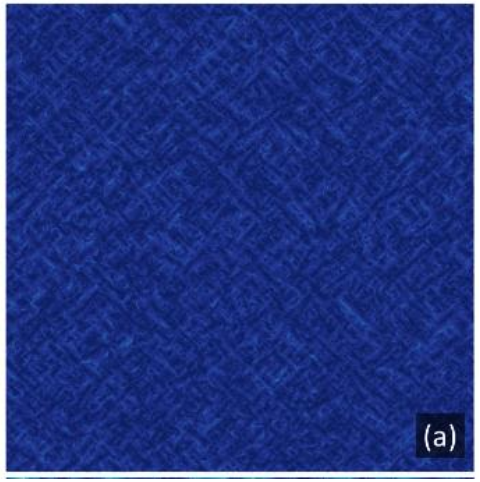
$$\langle \Sigma \rangle(\dot{\gamma}) - \Sigma_c = \dot{\gamma}^n$$

Scaling relation from avalanches: $\beta = \frac{1}{n} = \nu(d - d_f + z)$

Correlations with low driving



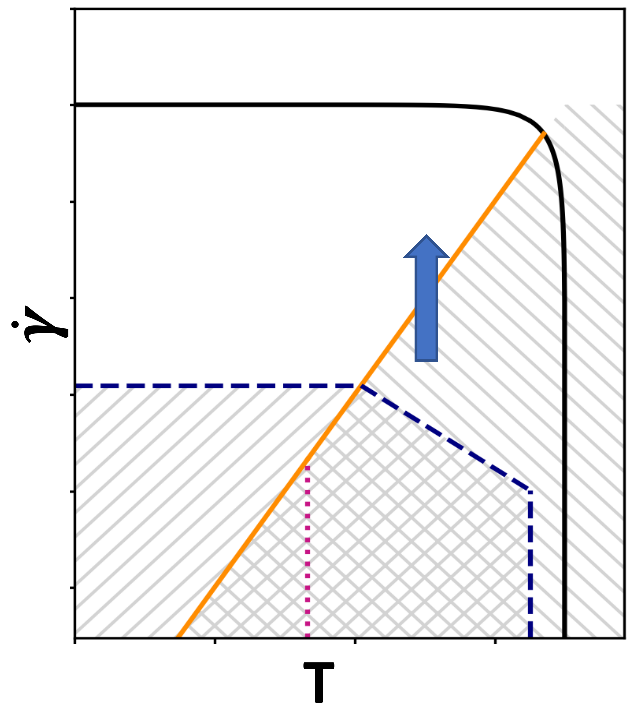
Correlations dimmish with high driving

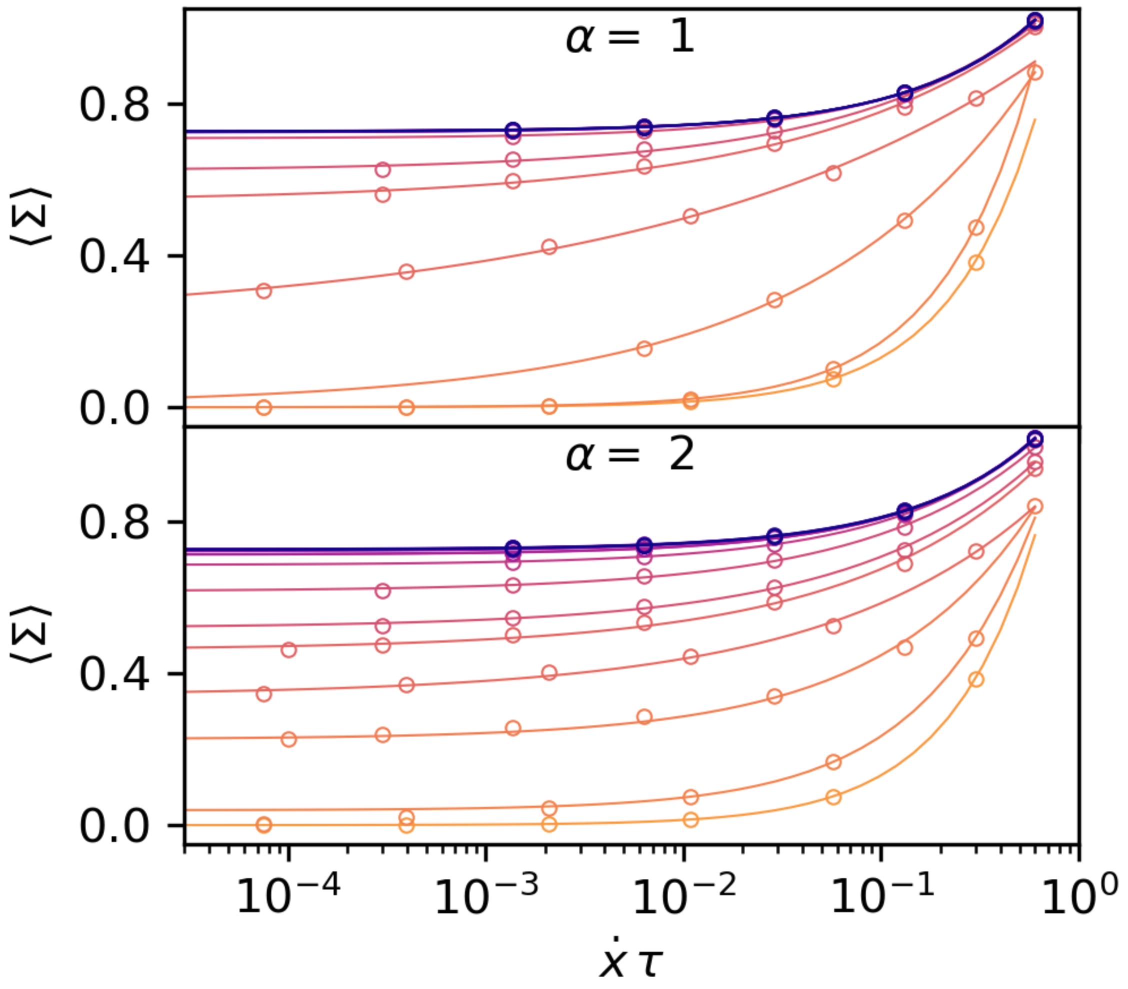


Results: Thermal Rheology

- Temperature reduces flow-stress

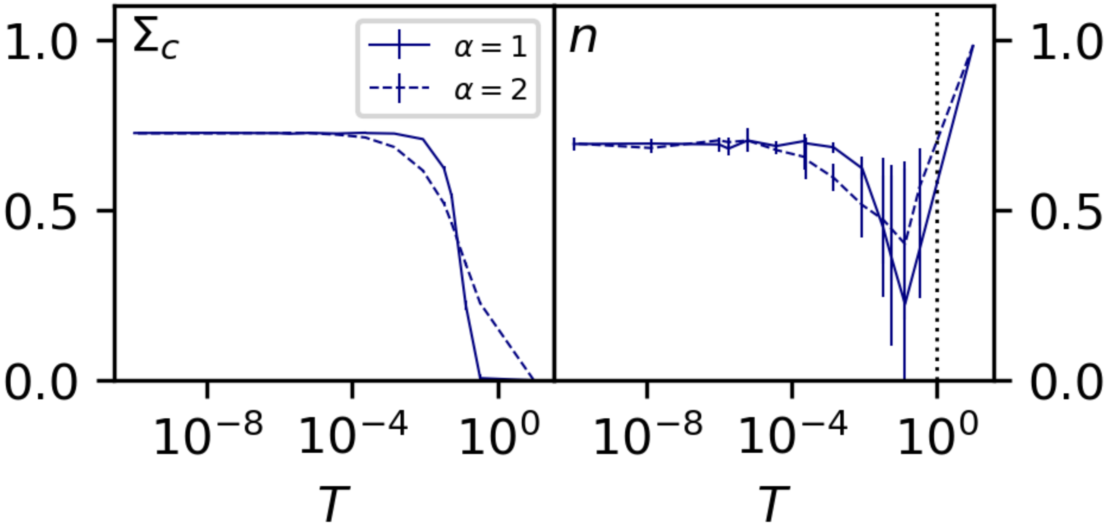
Crossing T,$\dot{\gamma}$ phase line

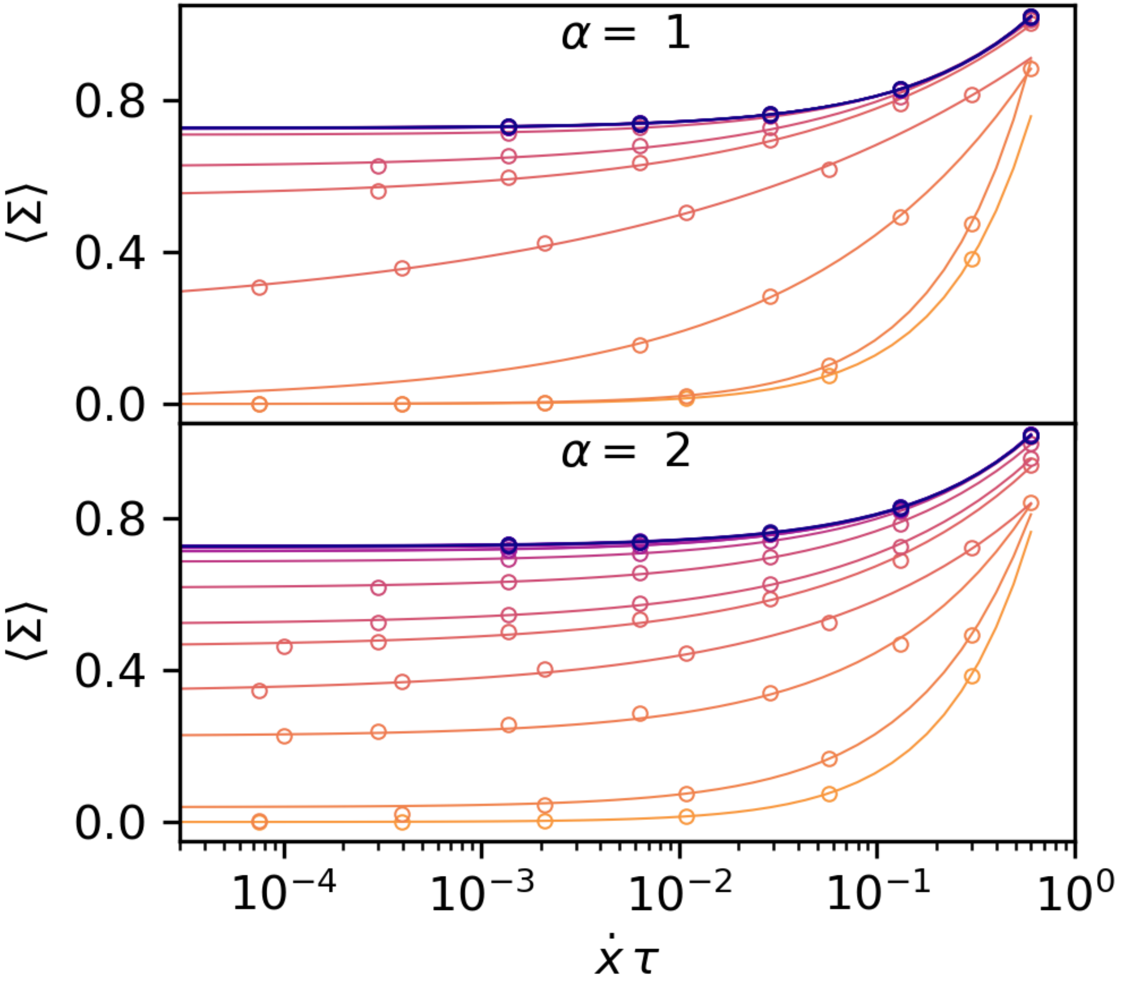




Results: Thermal Rheology

- Temperature reduces flow-stress
- Naïve Herschel-Bulkley fits
  $$\langle\Sigma\rangle(T) = \Sigma_c(T) + C\,\dot{\gamma}^n$$





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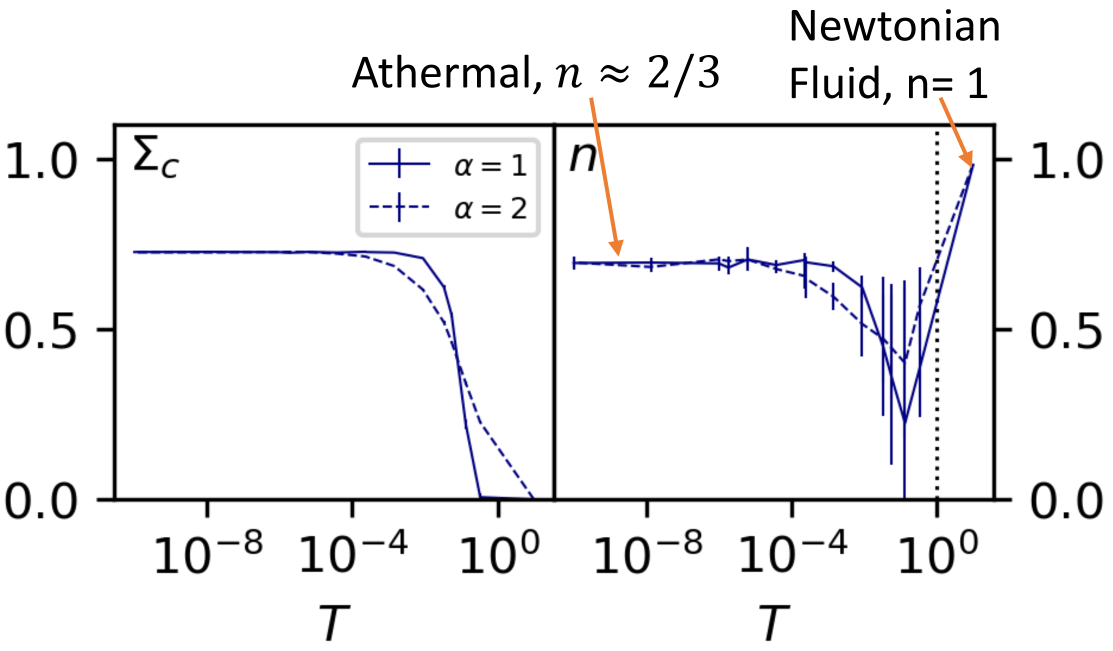

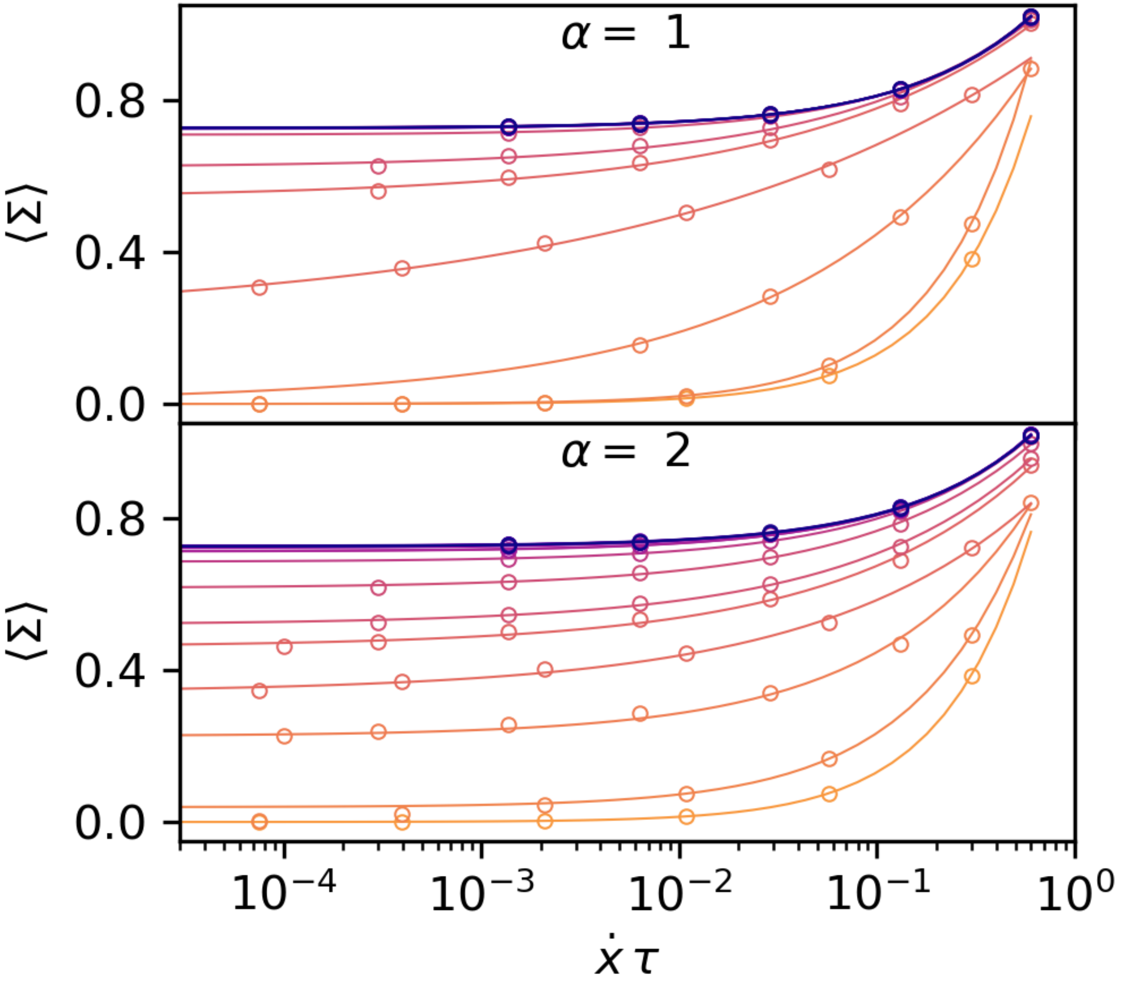

Results: Thermal Rheology

- Temperature reduces flow-stress
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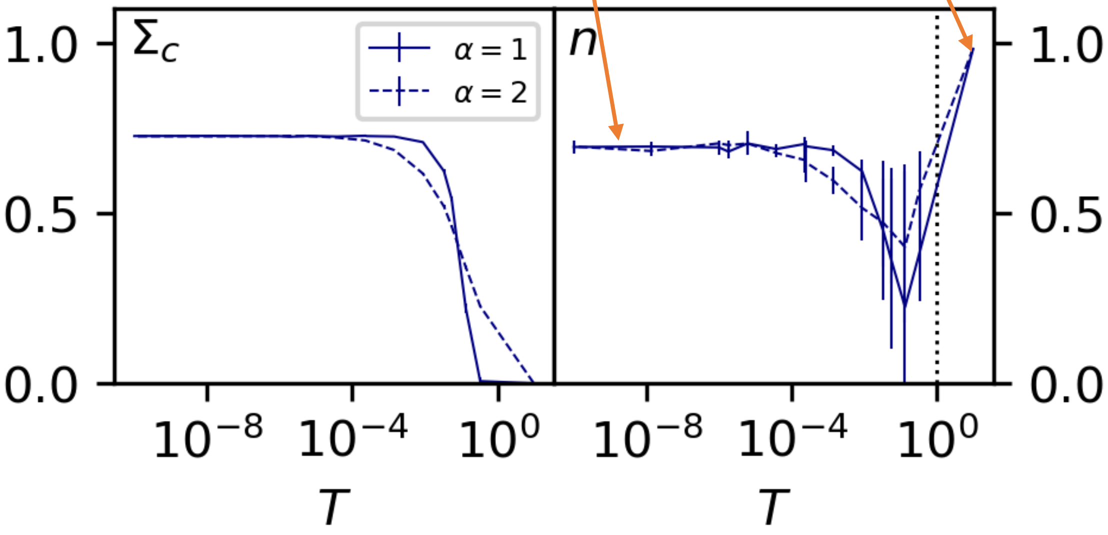
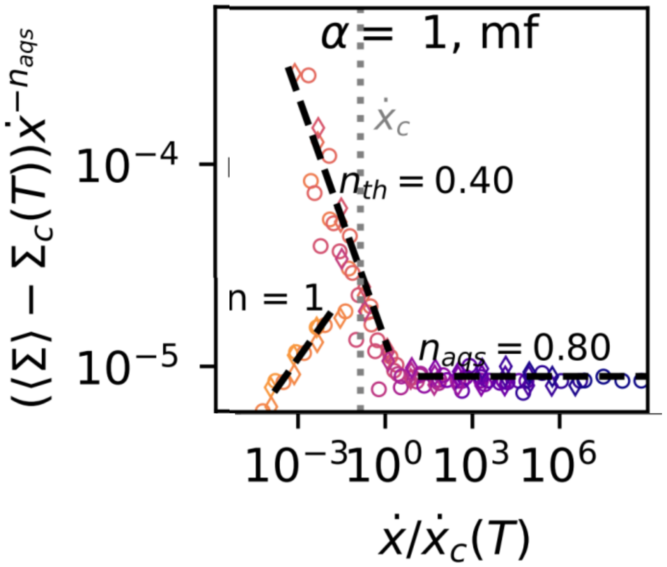
$$\langle\Sigma\rangle(T) = \Sigma_c(T) + C\,\dot{\gamma}^{n}$$

Athermal, $n \approx 2/3$

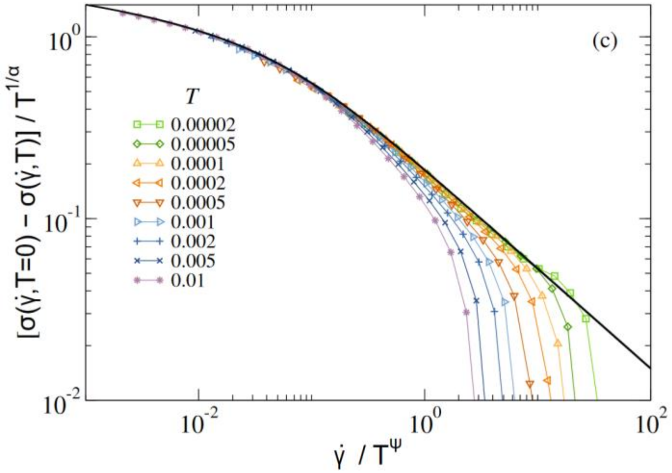
Newtonian Fluid, n= 1



A sharp exponent transition?



Or a logarithmic transition? (à la Dr. Jeudy's depinning talk)



See: Marko Popović et al. 2021 and Ezequiel Ferrero et al. 2021

Conclusions

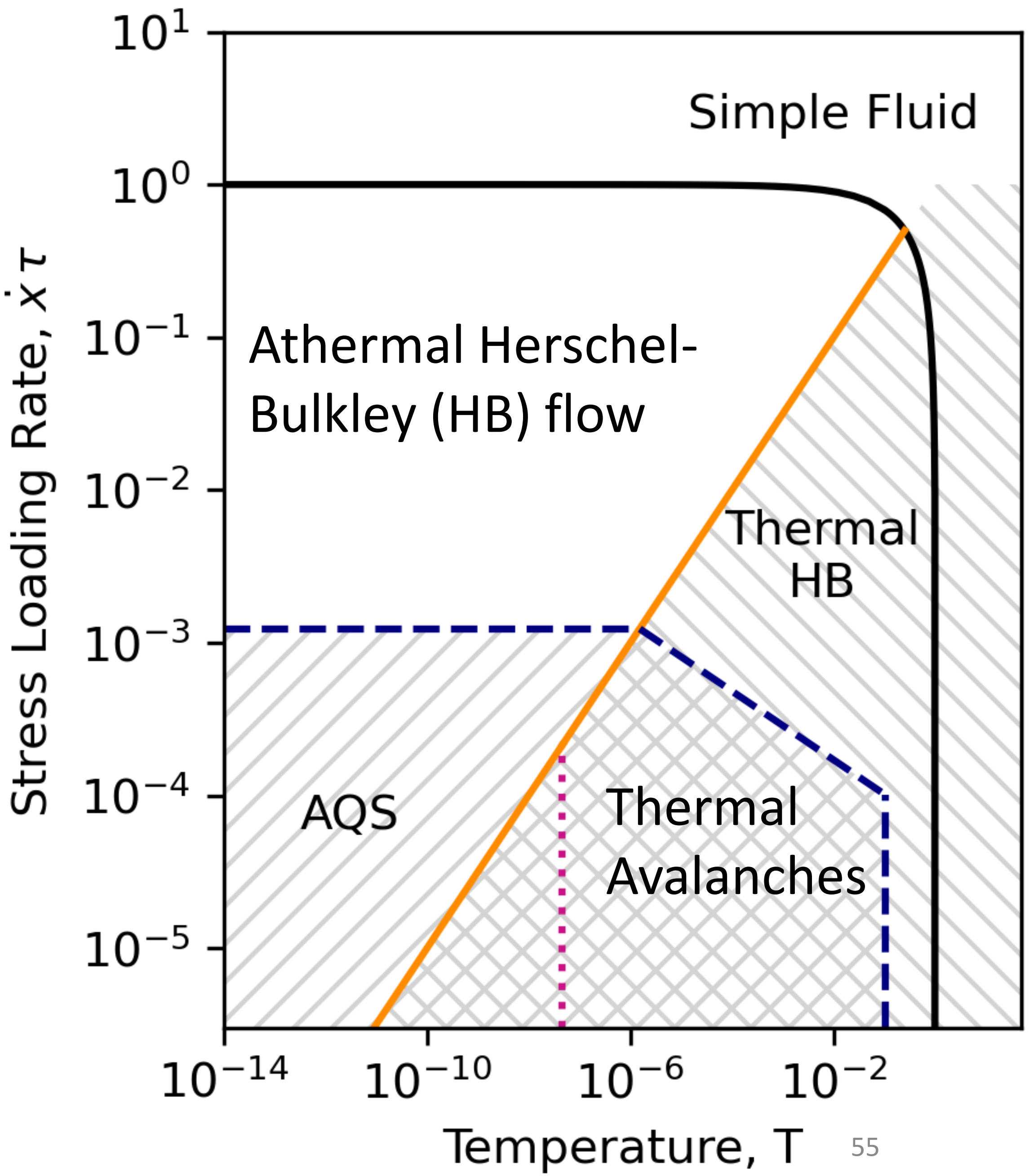
- Amorphous yielding is only SOC for (size dependent) slow driving and temperatures
- When do thermal effects appear?

$$\dot{\gamma} < \dot{\gamma}_c = \frac{1}{\tau} T^{\frac{1}{\alpha}}$$

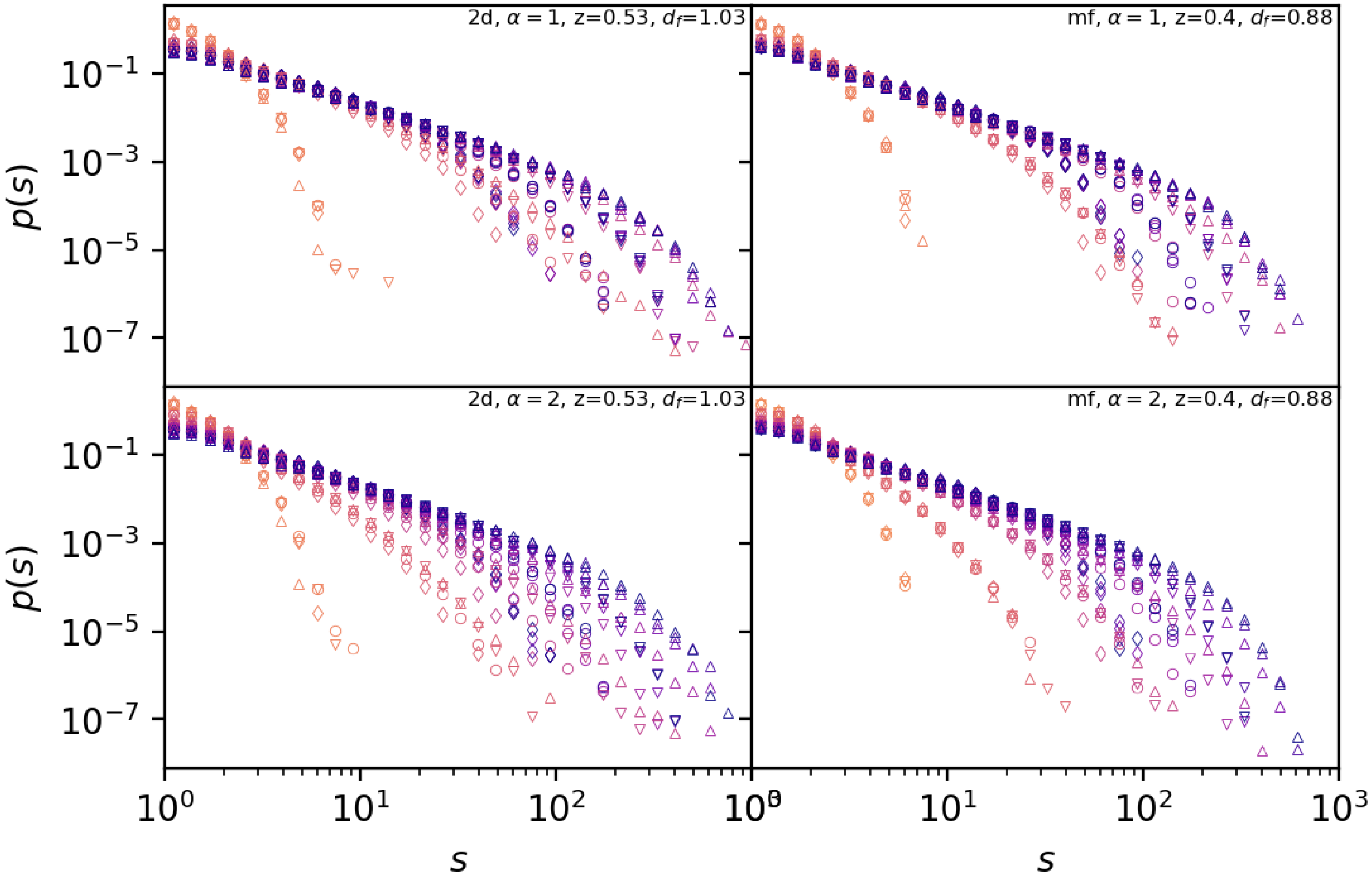
- Correlation length truncated by L or T
- Nontrivial T dependent rheology

For details, see: PhysRevE.106.034103
or say hi!

Phase Diagram of Thermal EPM



Avalanche size, system size, temperature



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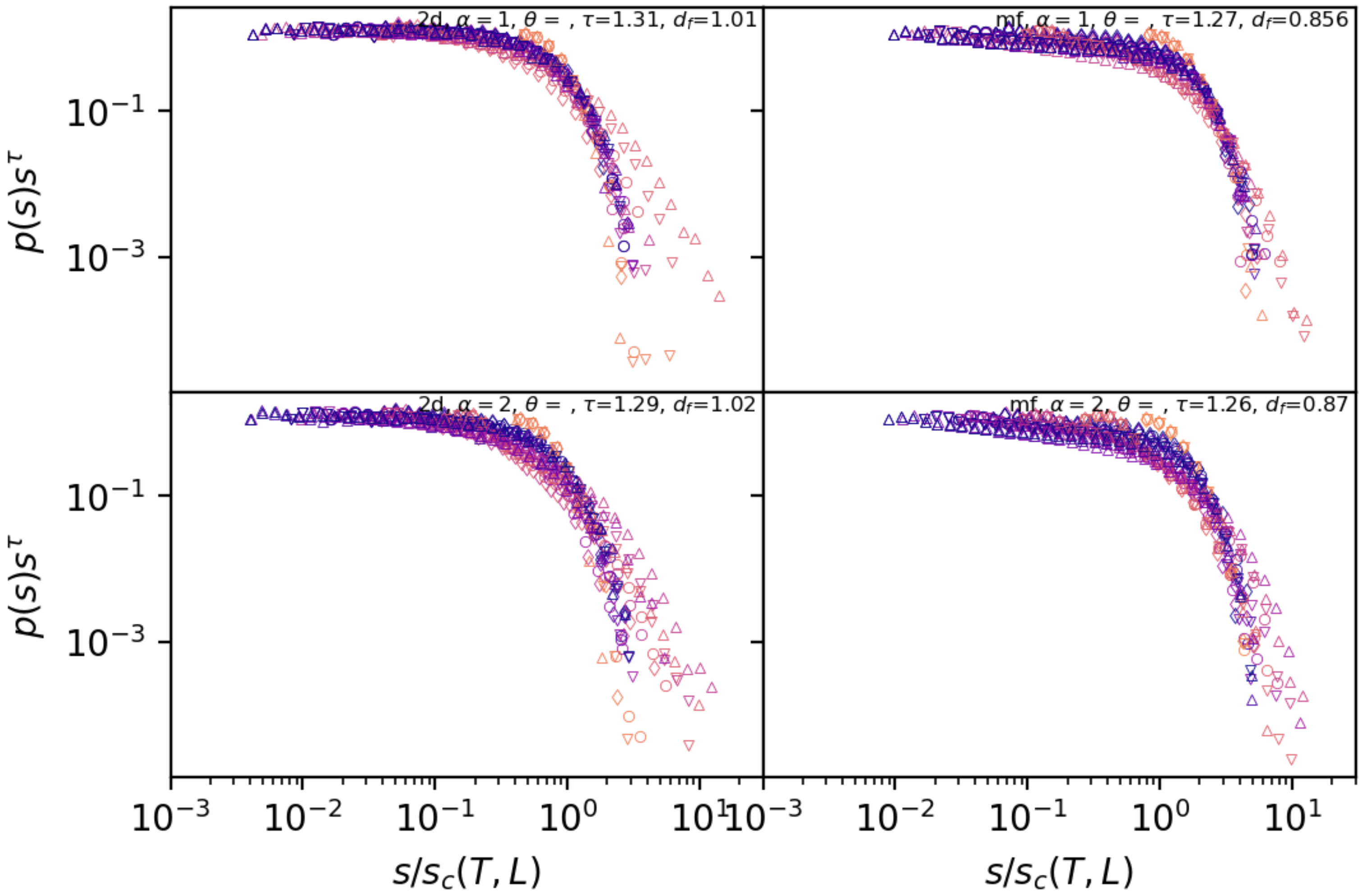
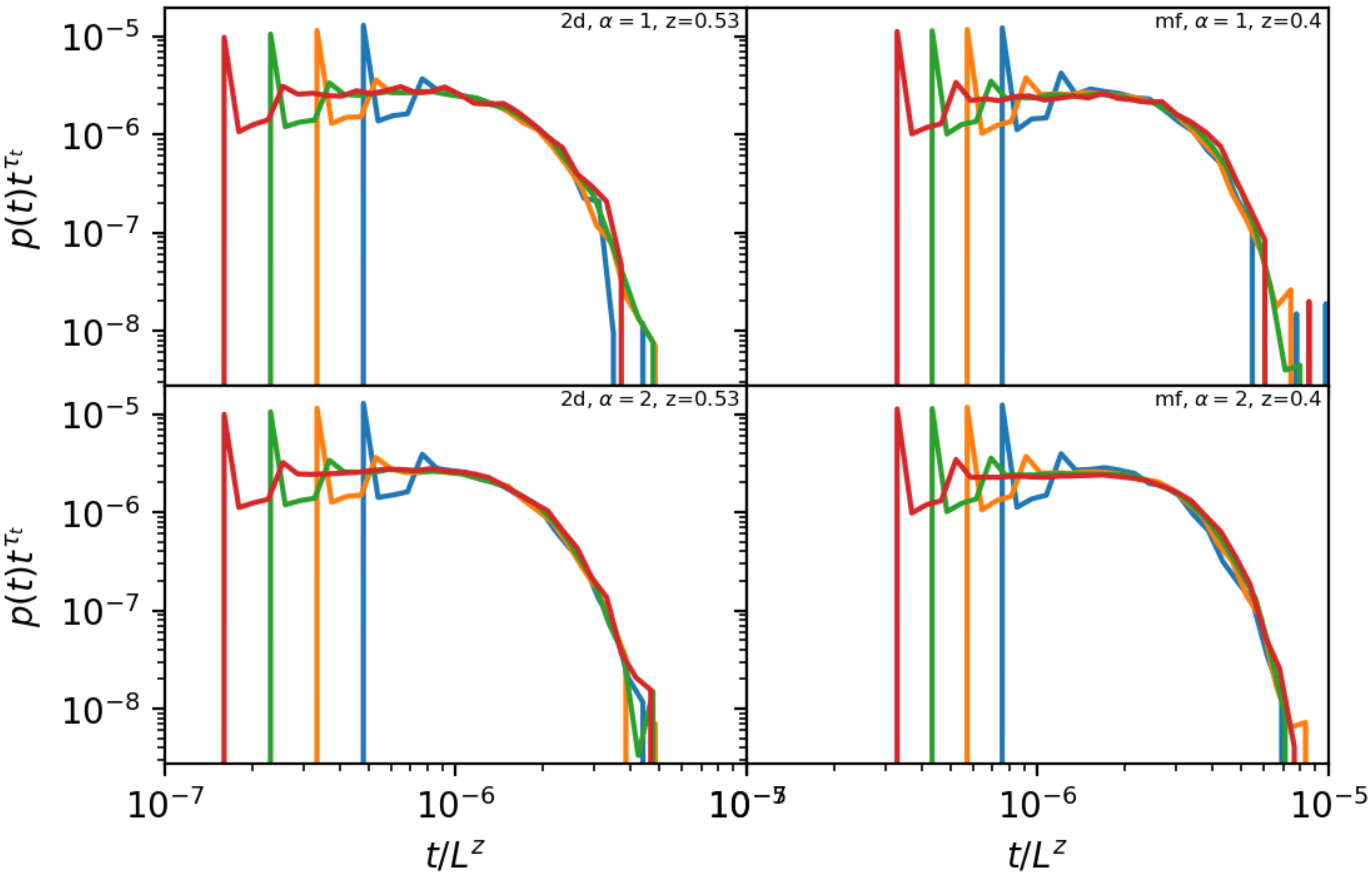
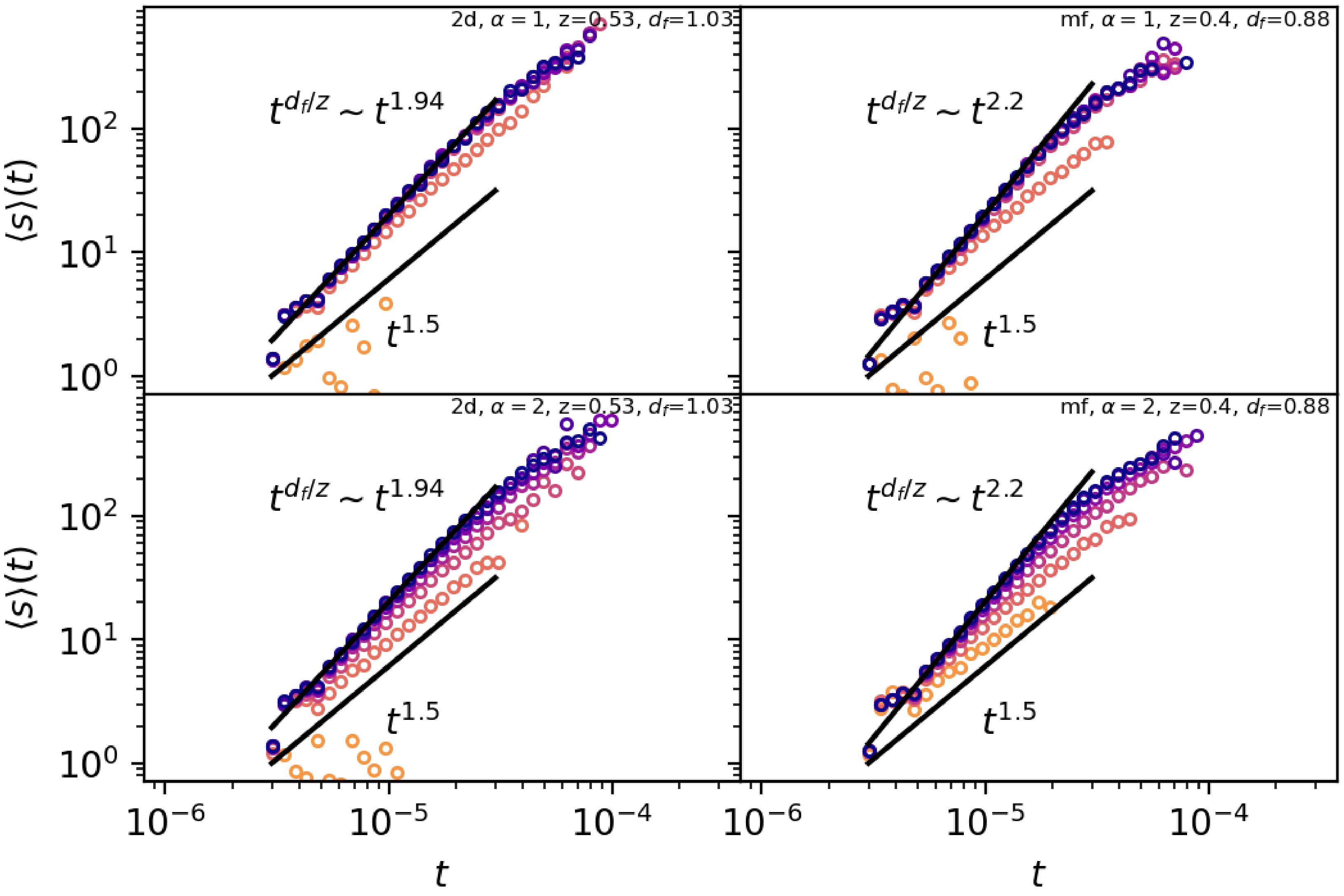


Avalanche time FSS

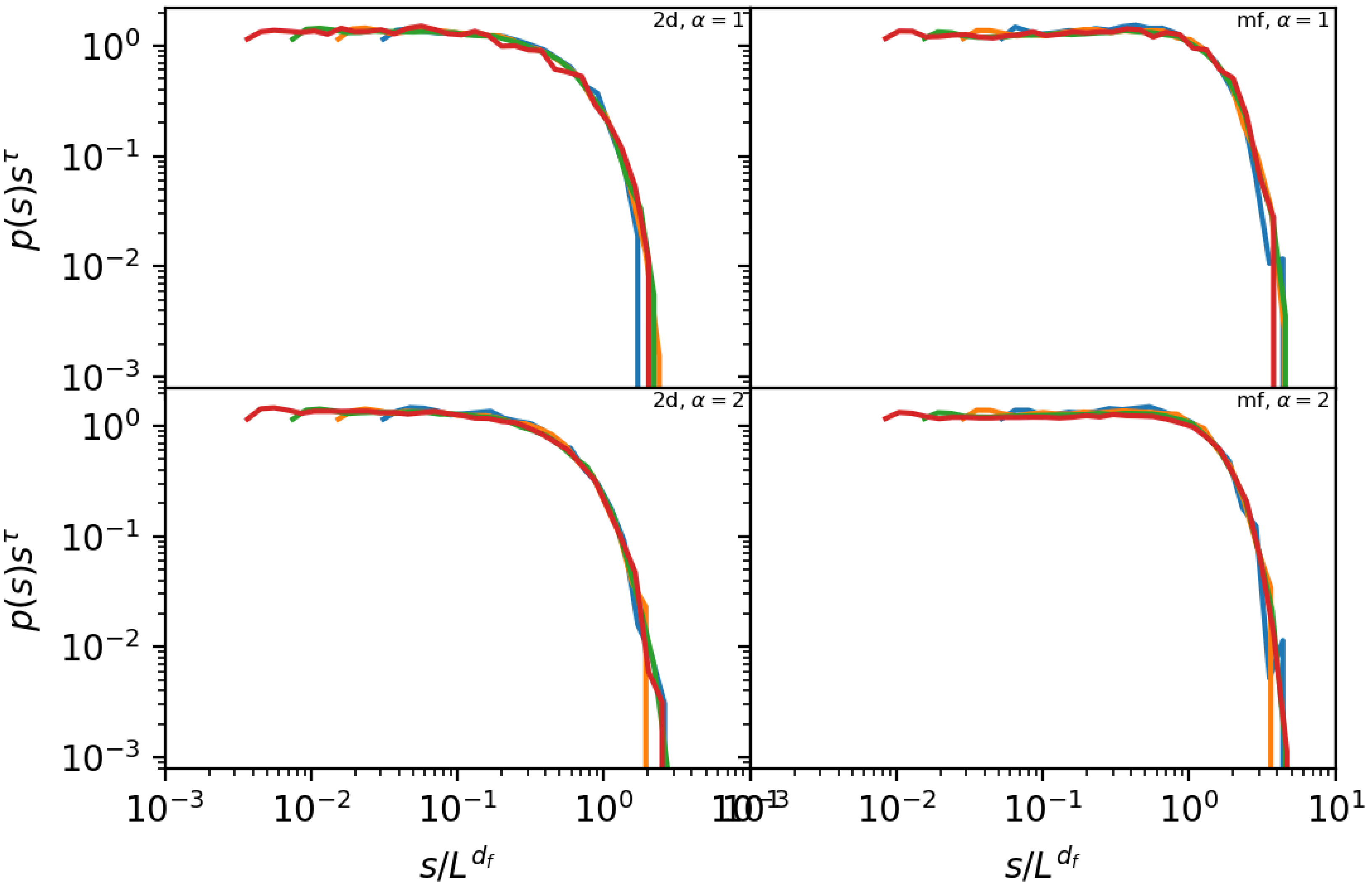
- $\tau_t \approx 2$



Size time scaling

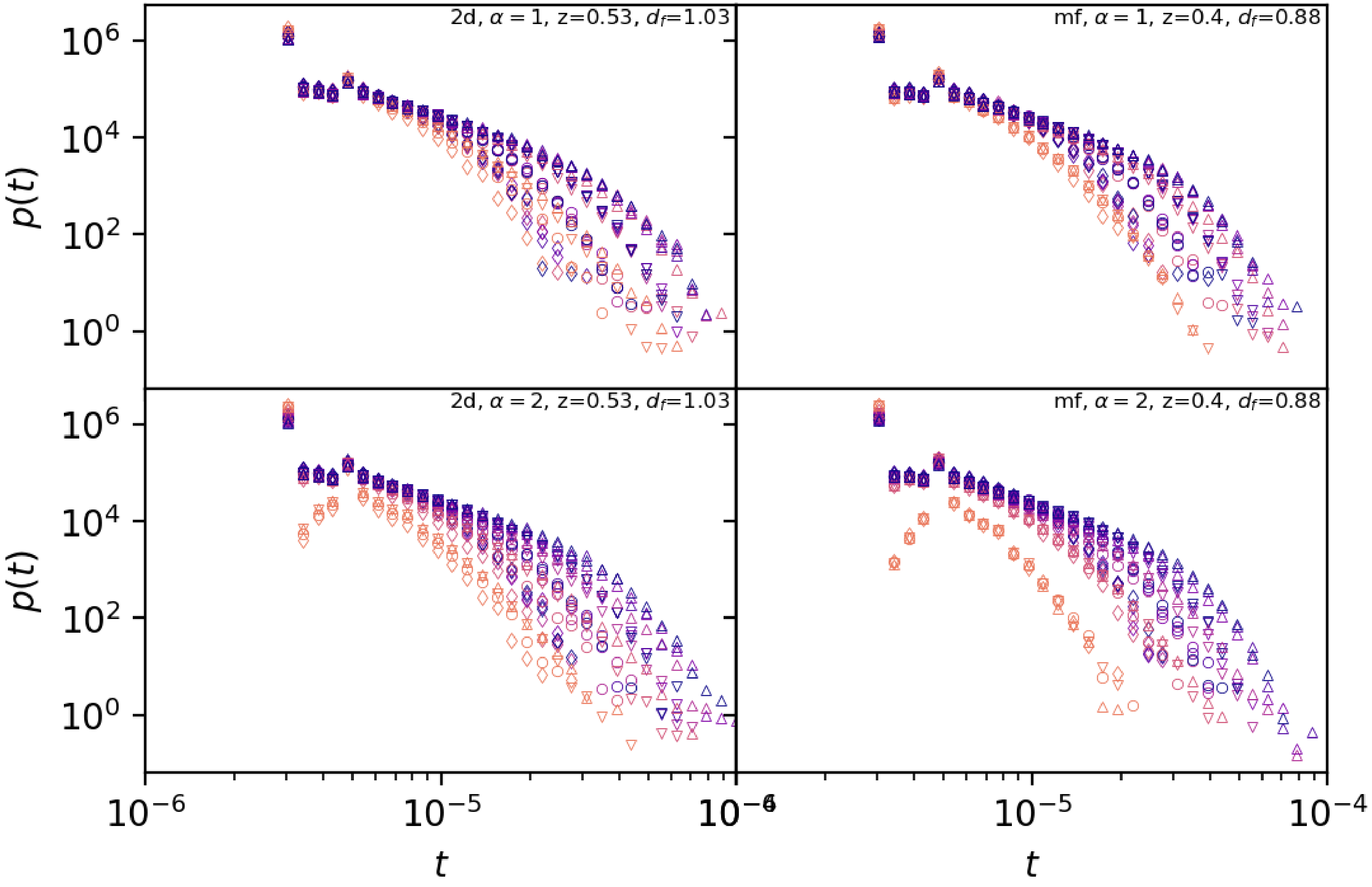


Size FSS

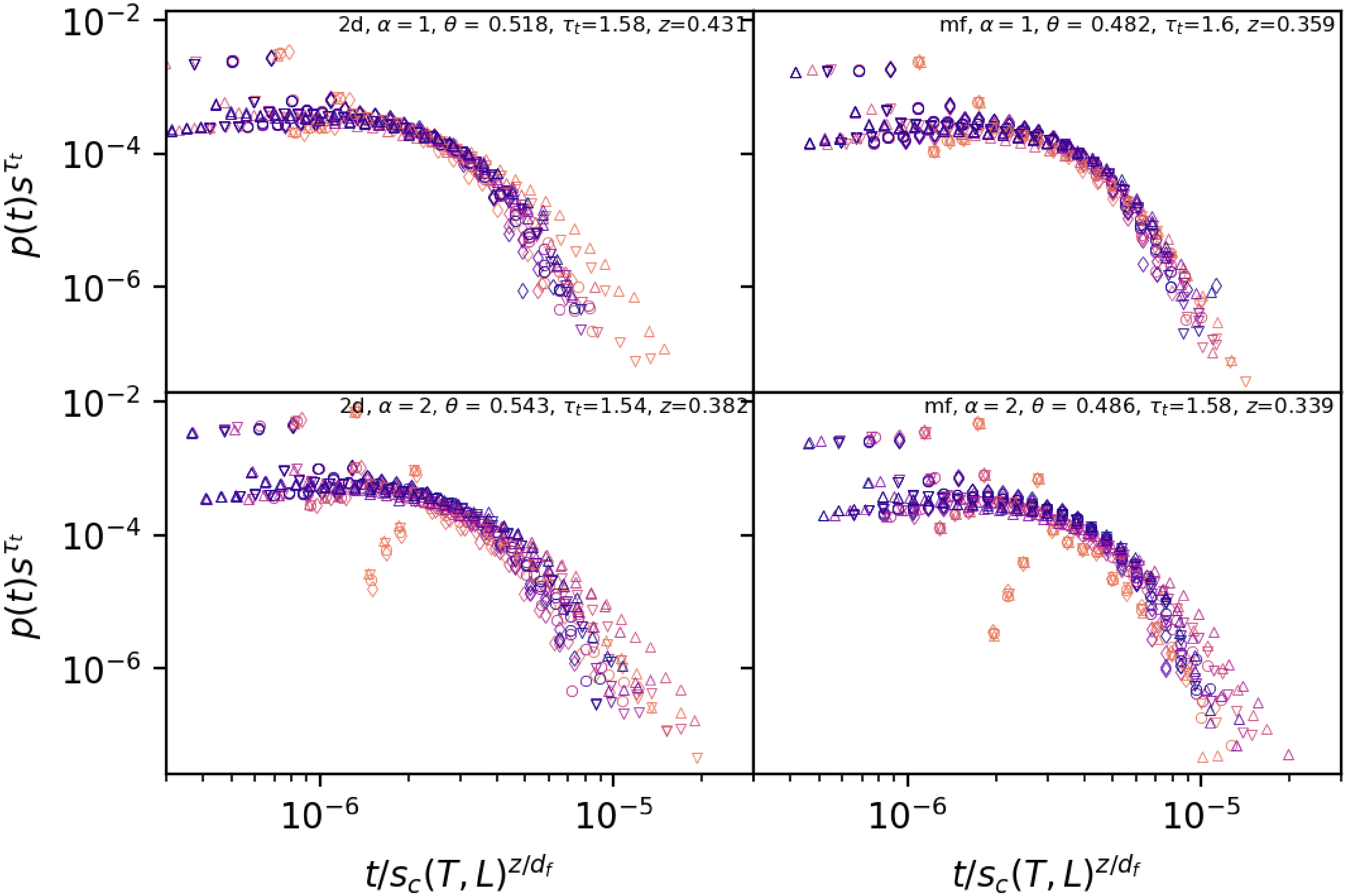


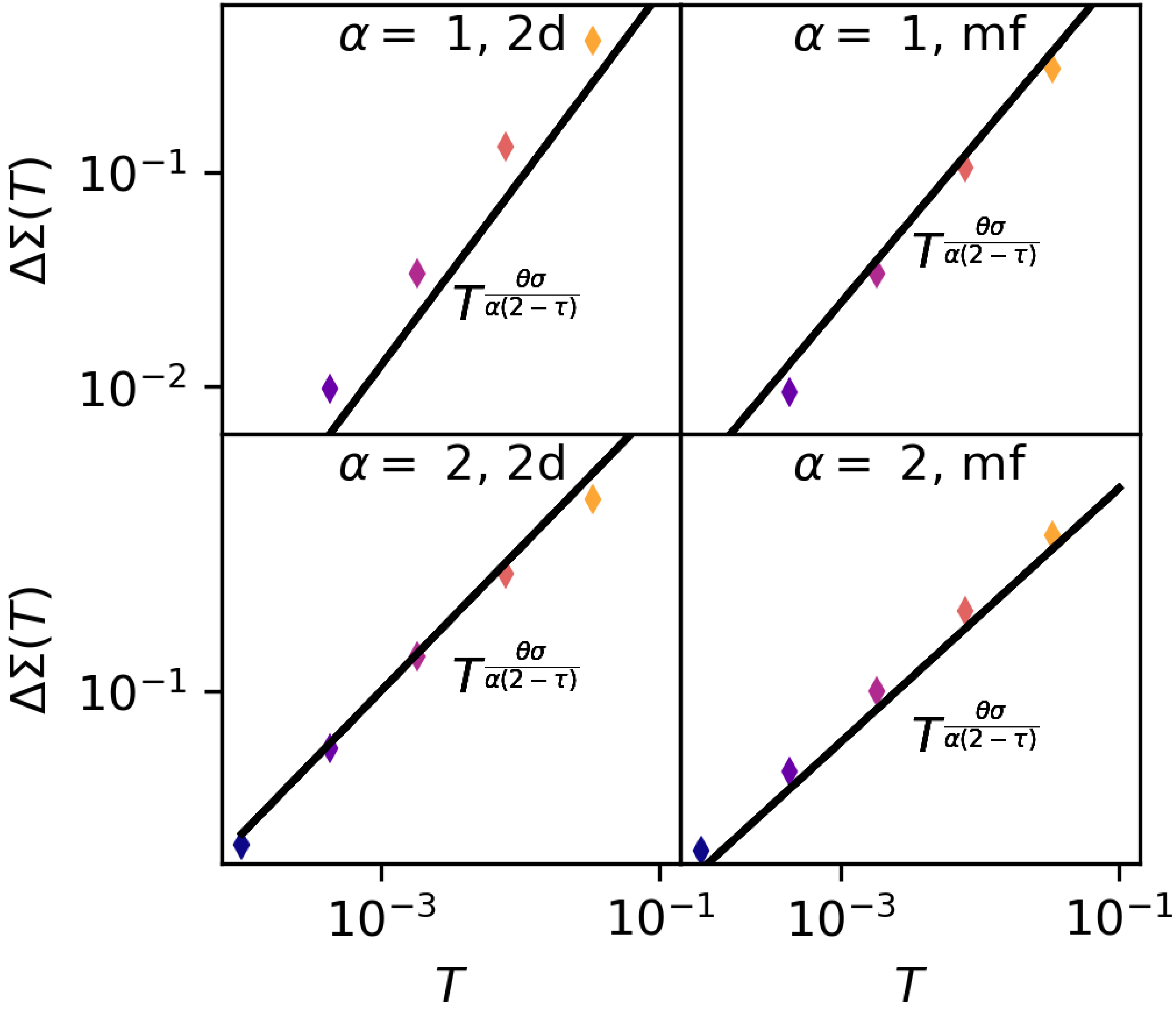
2d, $\alpha = 1$
mf, $\alpha = 1$
2d, $\alpha = 2$
mf, $\alpha = 2$
$p(s)s^{\tau}$
$p(s)s^{\tau}$
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
s/L^{d_f}
s/L^{d_f}

Avalanche duration and temperature



Temperature and duration collapse





α = 1, 2d
α = 1, mf
α = 2, 2d
α = 2, mf
$T^{\frac{\theta\sigma}{\alpha(2-\tau)}}$
$\Delta\Sigma(T)$
10^{-1}
10^{-2}
10^{-3}
T