

Richness and complexity of slip events at a frictional interface

J.F. Molinari and Thibault Roch (lsms.epfl.ch)

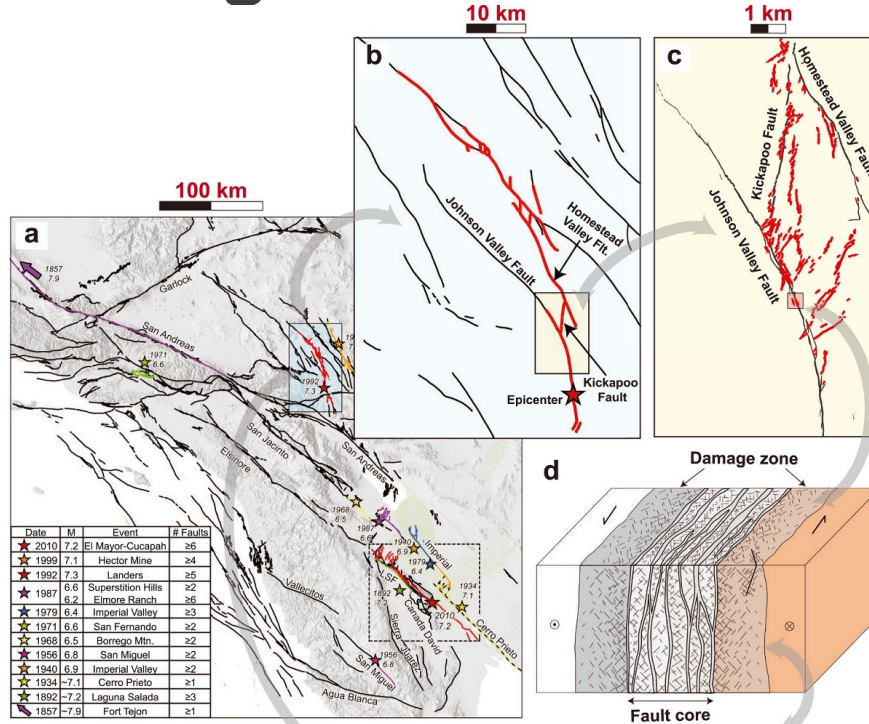
and

F. Barras (Univ Oslo)

E. Bouchbinder and M. Aldam (Weizmann)

E. Brener (Juelich)

Earthquake physics: Heterogeneities across scales; complexity

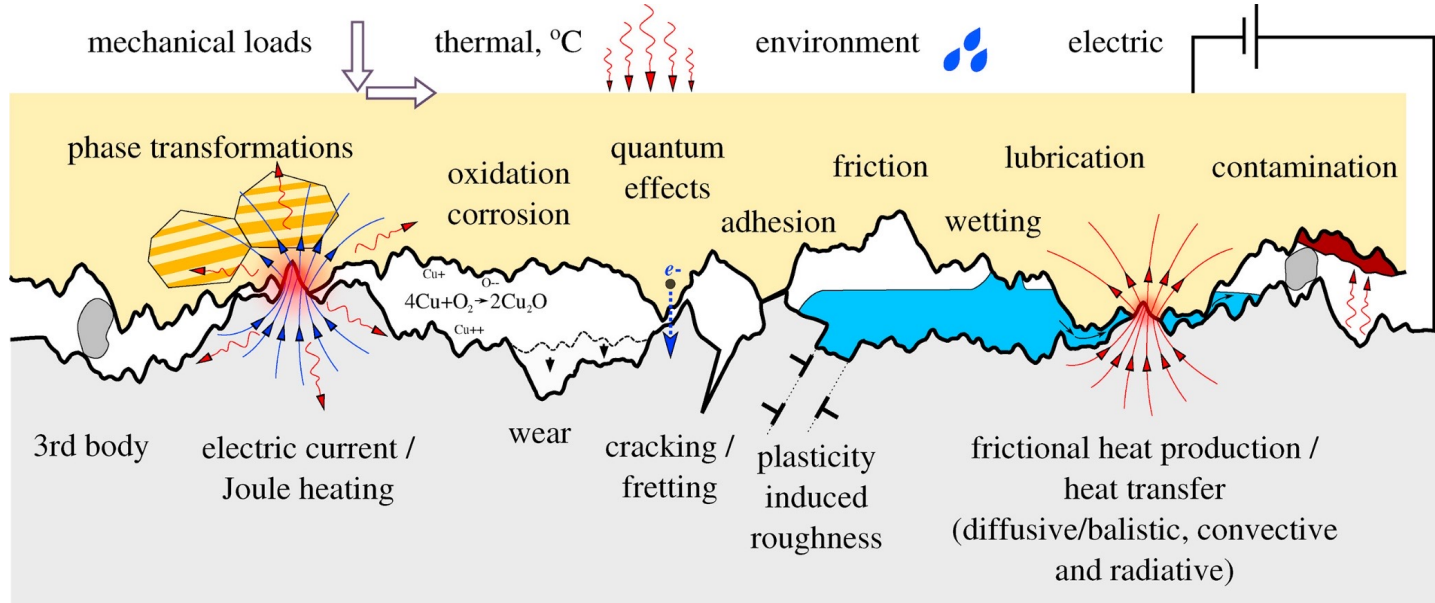


Diversity of mechanisms:

- On fault and off fault damage
- Gouge
- Fluids, poro-elasticity
- Flash heating
- Thermal pressurization
- ...

Okubo et al., J. of Geophys. Res. Solid Earth (2019)

Complexity in tribology



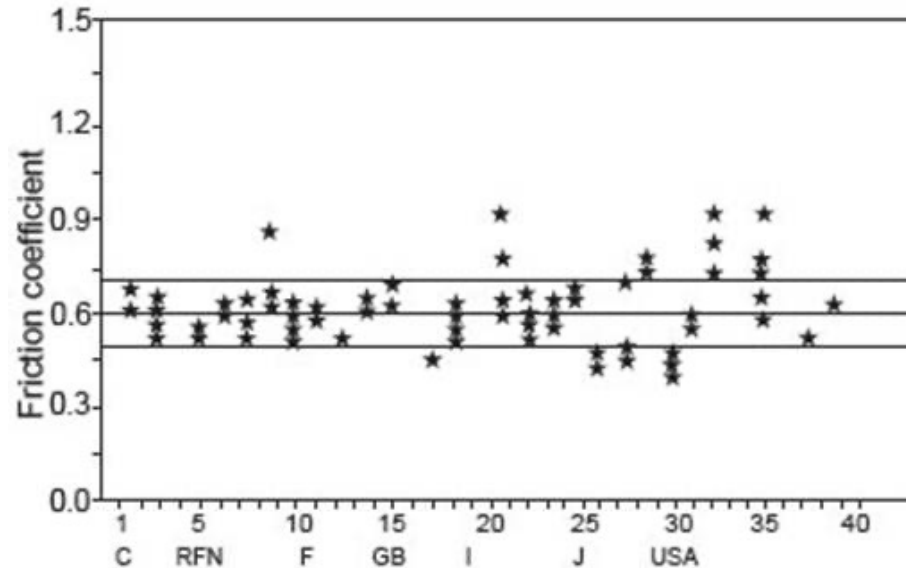
Review paper: *Vakis et al., Tribology International, 2018*

Meng and Ludema, 1995: 300 equations on friction and wear (1957-1992)

some with up to 25 material parameters and fit constants

VAMAS report. Vamas.org

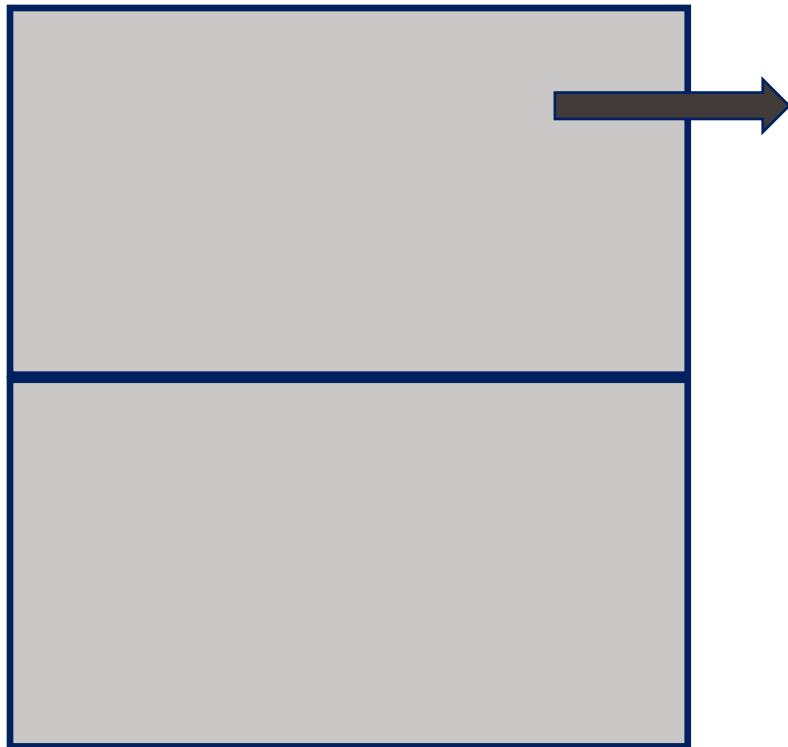
“...the friction between identical steel and the aluminum oxide samples was tested in various labs in the world (VAMAS) ...surfaces of samples had the same roughness parameters, the ambient of every test was similar (special air conditioned rooms); the load applied on samples (pressures) and sliding speed were the same”.



Jan van de Snepscheut:

“In theory, there is no difference between theory and practice. But, in practice, there is.”

Overview of talk; very simple model for sliding dynamics



Ingredients:

- 2D finite or semi-infinite elastic bodies
- Rate and state friction at the interface
- Stress or velocity controlled loading
- PBCs on the sides
- Homogeneous properties
- Elastodynamics
- Yet, richness and complexity will emerge:
- Elasticity, Interaction, Disorder

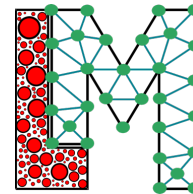
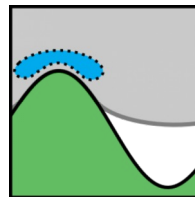
- Track record of development of numerical methods and open-source software (all on GitLab; HPC)

- Akantu

- Cracklet

- Tamaas

- Libmultiscale



- Akantu:** general purpose FE software (statics and dynamics, contact detection, cohesive elements, non-local continuum damage, phase-field fracture); costly (mesh everywhere; reduced time step compared to Cracklet); but finite boundaries (good control of BCs)
- Cracklet:** spectral boundary element code for elastodynamics of cracks and sliding friction (Geubelle and Rice 1995; Breitefeld and Geubelle 1998); very fast (discretization of interface only; semi-infinite elastic bodies in contact)

Past and current LSMS PhD theses on frictional rupture



Prof. David Kammer,
ETHZ, Switzerland
PhD 2014



Dr. Fabian Barras,
Univ. Oslo, Norway
PhD 2018



Thibault Roch,
EPFL
PhD June 2023



Roxane Ferry,
EPFL
Sept 2022 -

- Dynamic stress drops:
 - When friction can be explained with tools of fracture mechanics
 - And when friction is friction
- Boundary conditions matter: velocity versus stress controlled loading
 - Pulse-like versus crack-like frictional rupture
- Emergence of statistical complexity in simple systems with **no** heterogeneities

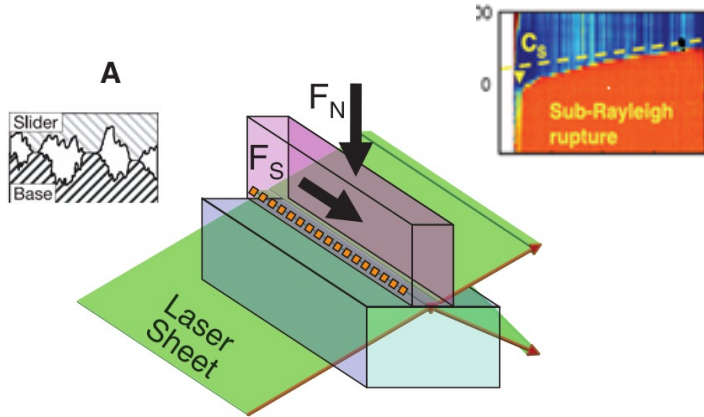


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Svetlizky, Fineberg, *Nature*, 2014

Ben-David, Cohen, Fineberg, *Science*, 2010

Kammer, ..., Fineberg, *Science Advances*, 2018

Kammer, Radiguet, Ampuero, Molinari, *TL*, 2015

Xia, Rosakis, Kanamori, *Science*, 2004

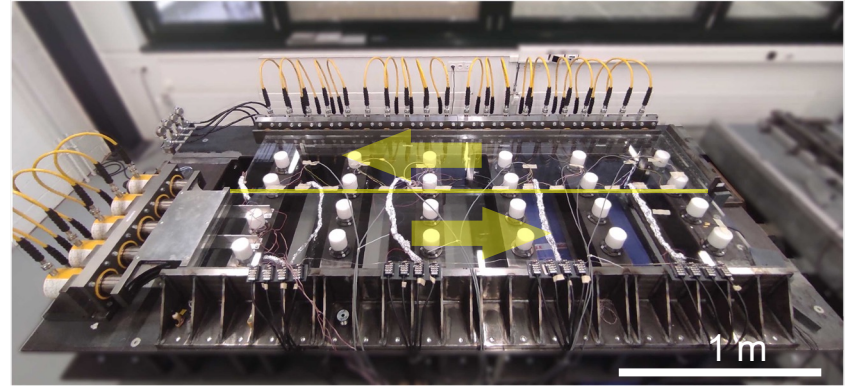
Rubino, Lapusta, Rosakis, *Nature*, 2022

Pagialunga, ..., Violay, *EPSL*, 2022

Cebry, ..., McLaskey, *Nat. Comm.*, 2022

Yamashita, Fukuyama, ..., *Nature*, 2015

And others...

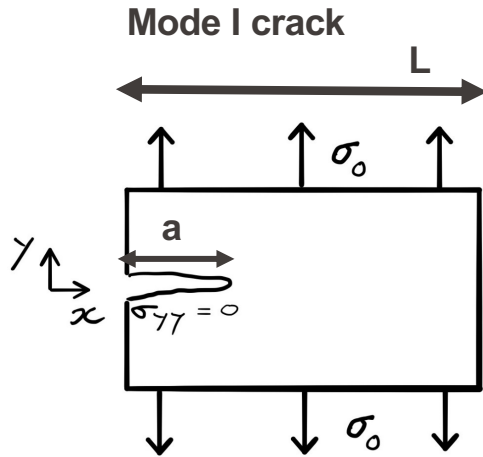


Pagialunga, Passelegue, Violay, in preparation

**Evidence from lab earthquakes (and seismology data) :
frictional cracks explained by LFM**

But why crack-like properties emerge is not clear

Analogy to fracture mechanics?



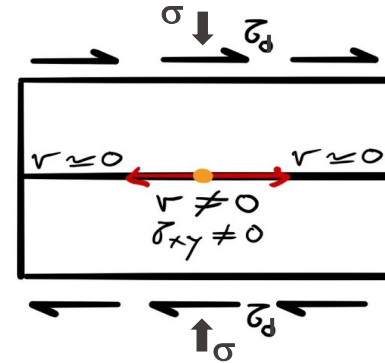
Stress drop on crack faces (zero stress)

⇒ stress singularity ($1/r^{1/2}$) at crack tip

Stress intensity factor: $K_I \propto \sigma_0 \sqrt{a}$

Energy balance: $G = G_c$ $G = \frac{K_I^2}{E}$

Mode II (or III) frictional rupture



Creation of new surfaces ?

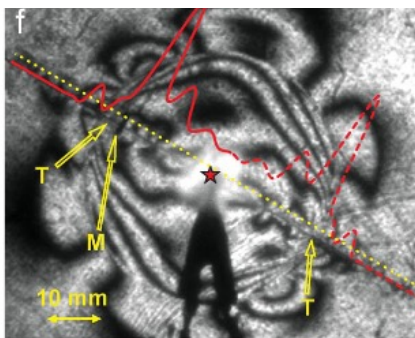
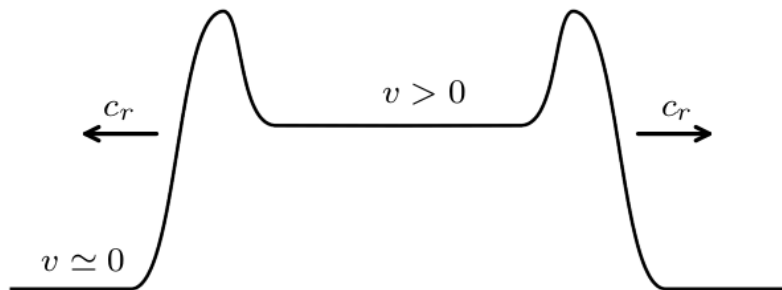
Stress drop on sliding interface?

Friction: interface carries stress

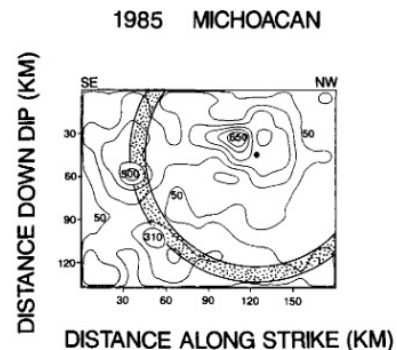
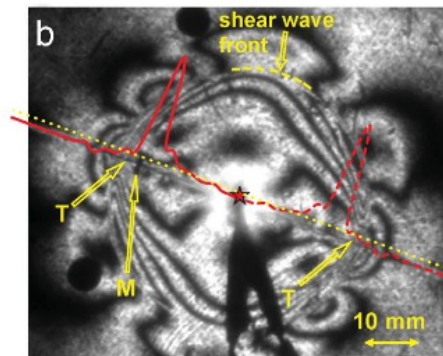
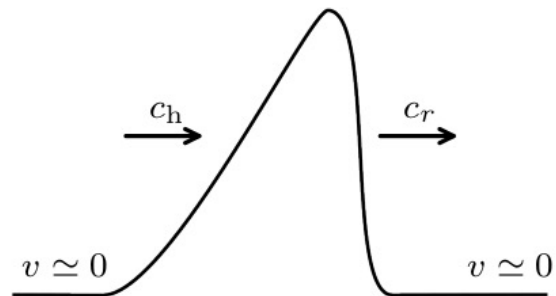
Stress singularity? Energy balance?

Fracture energy versus breakdown work?

Crack like



Slip pulse



Lu, Lapusta, Rosakis, PNAS, 2007

Heaton, Phys. Earth Planet. Int., 1990

Ingredients for elastodynamics

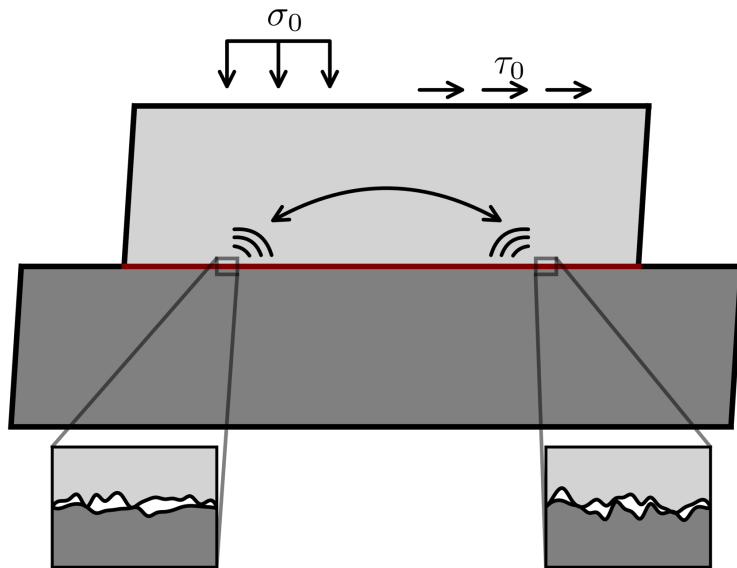
Linear elastic **finite or infinite** continuum, **velocity or stress controlled loading**

2D mode III (plane strain) or mode II (plane stress)

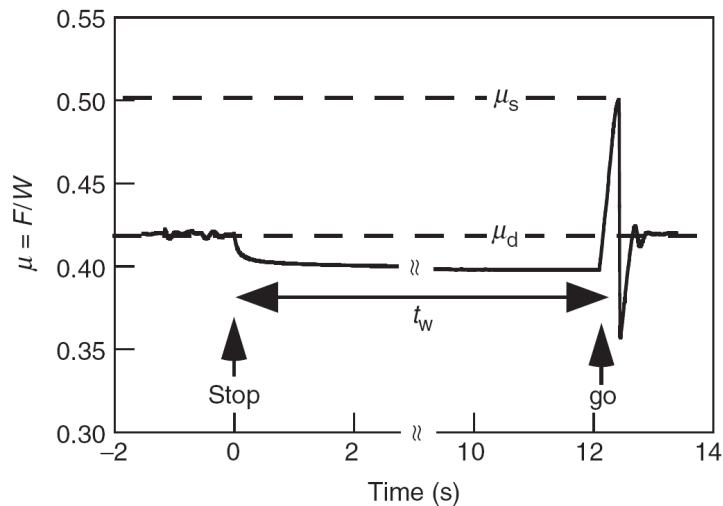
Perturbation to initiate rupture front (Brener, Aldam, Barras, Molinari, Bouchbinder, *PRL*, 2018)

Rate and State friction at the interface

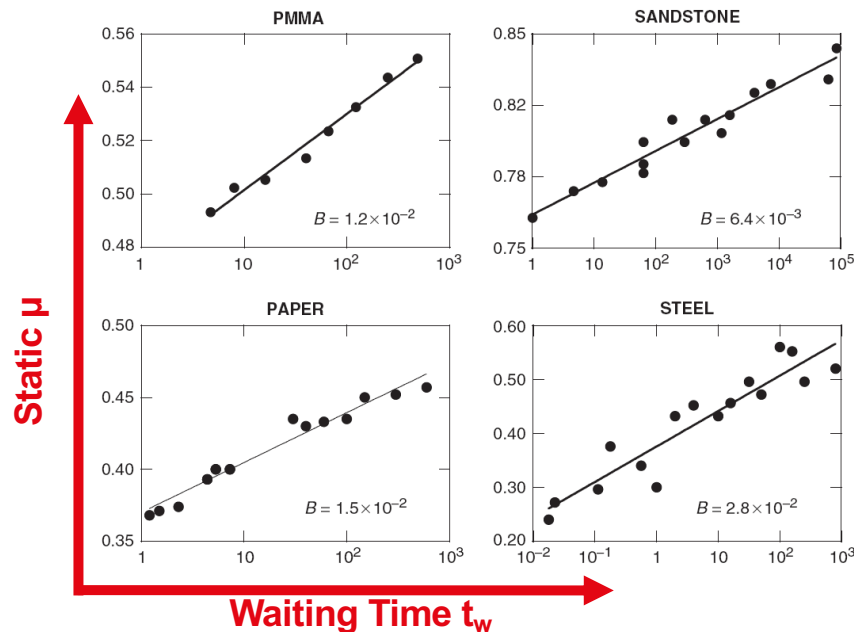
Interplay between bulk and interface properties



T. Baumberger, C. Caroli, 2006 (review paper)

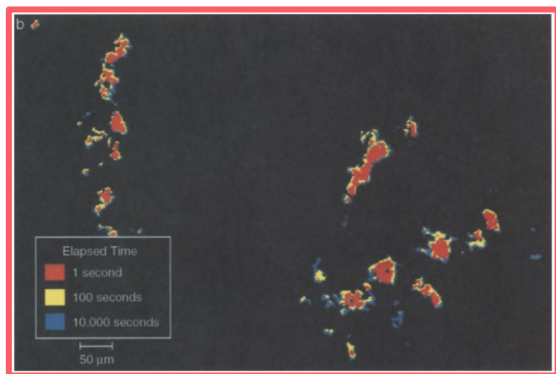


$$B = \frac{d\mu_s}{d(\ln t_s)}$$

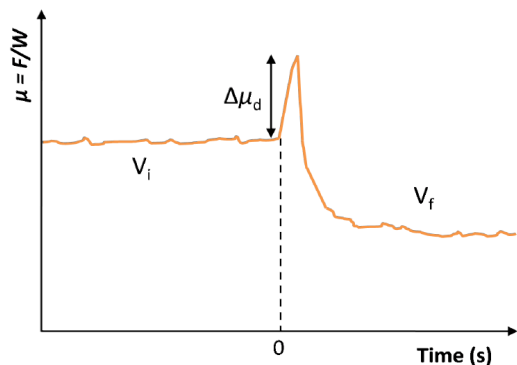


Micro contacts, state and rate parameter

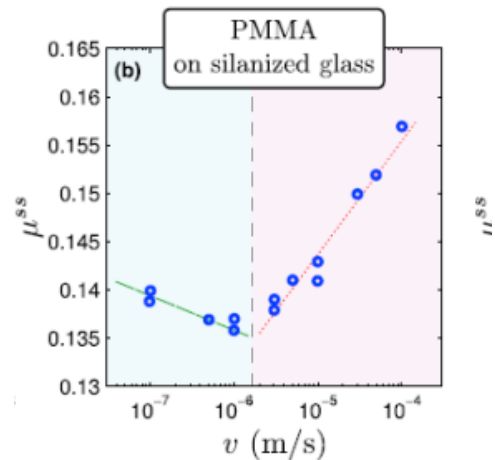
Dieterich & Kilgore (1994)



Dieterich (1979)



Bar-Sinai et al., JGR (2014)



Phenomenological R&S friction: Dieterich (1979), Rice and Ruina (1983), ...

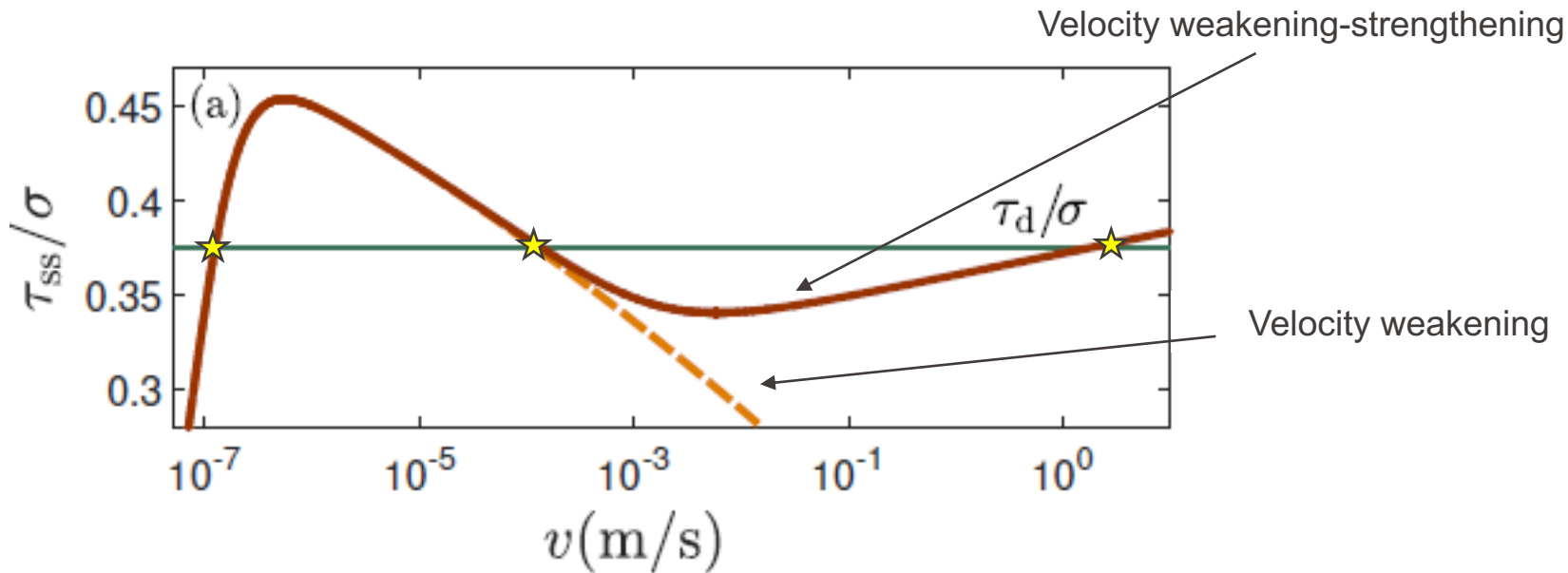
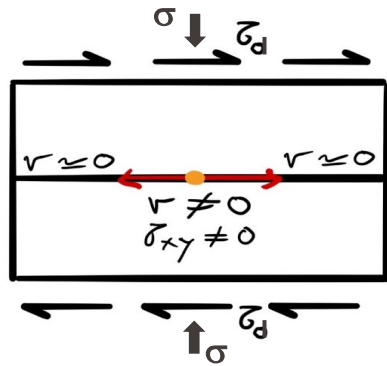
$$c_f(v, \phi) = c_f^0 + A \ln \left(1 + \frac{v}{v^*} \right) + B \ln \left(1 + \frac{\phi}{\phi^*} \right)$$

$$\dot{\phi} = 1 - \frac{v\phi}{D}$$

Rate and state friction

Load-controlled system

$$c_f \left(v = v^{SS}, \phi = \frac{D}{v^{SS}} \right)$$



Dynamic stress drop (for fast rupture)

Barras et al., PRX, 2019

At finite times, before reflections from boundaries, $\omega(H/c_s)$, temporary steady state:

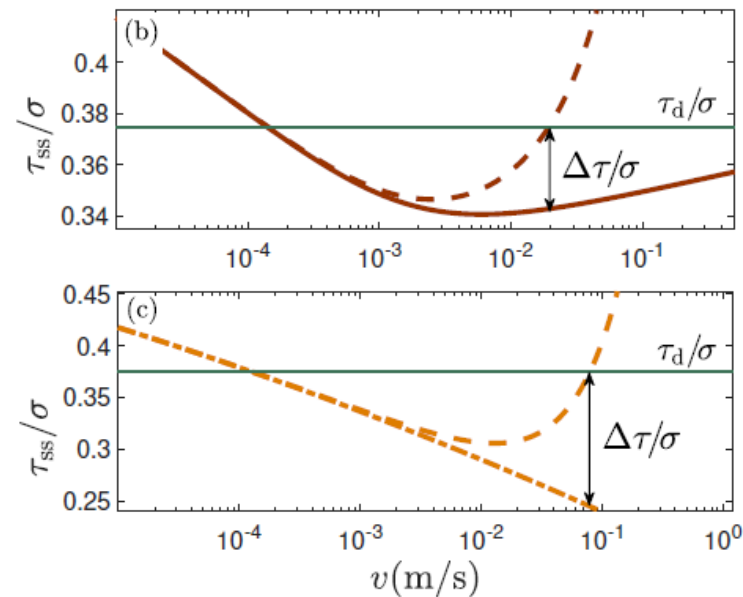
$$\tau_{SS}(v_{res}) + \frac{\mu}{2 c_s} (v_{res} - v_0) = \tau_d$$

$\Delta\tau$ (radiation damping)

Effective steady state friction curves
(dashed curves)

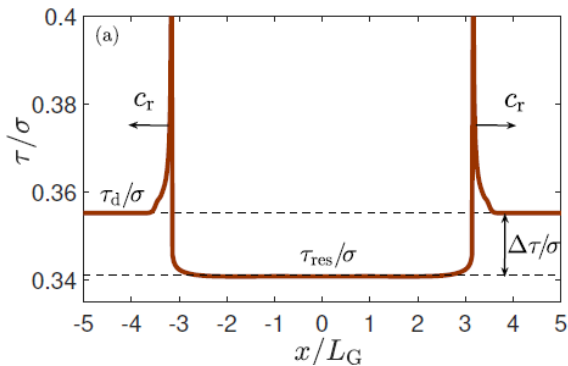
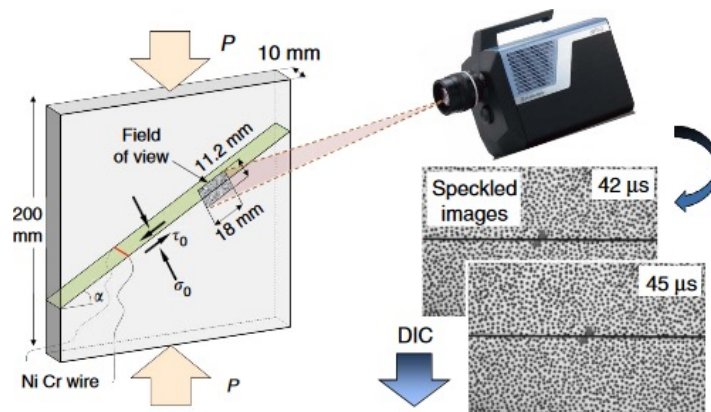
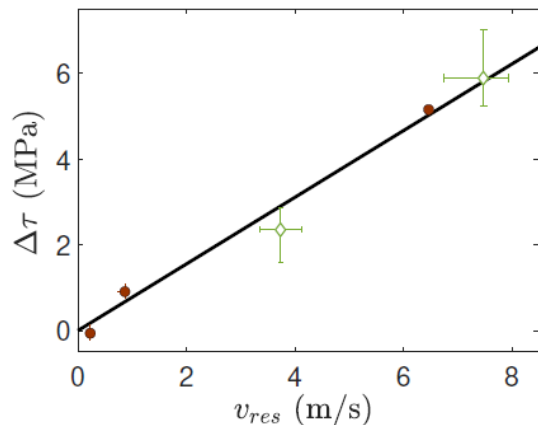
Give rise to a temporary stable sliding

In presence of dynamic stress drop:
«Friction is fracture»

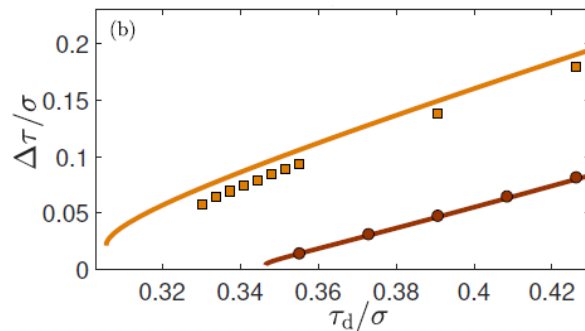


Theory matches exp and simulation data

Rubino, Rosakis, Lapusta, Nat. Comm, 2017



Barras, et al.,
PRX, 2019

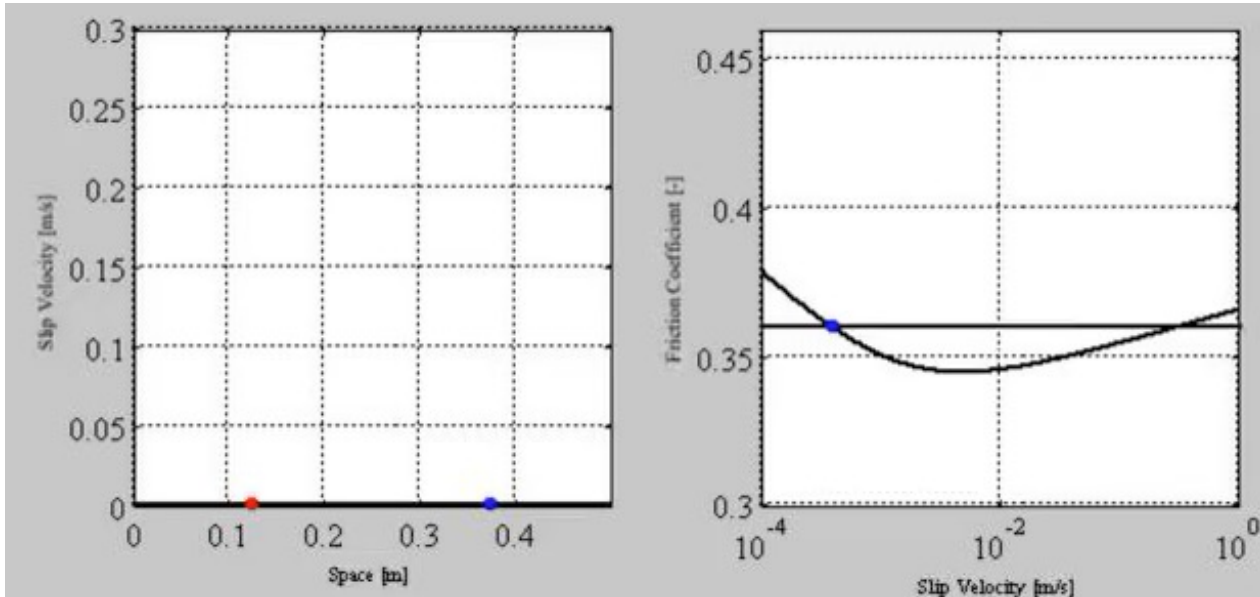
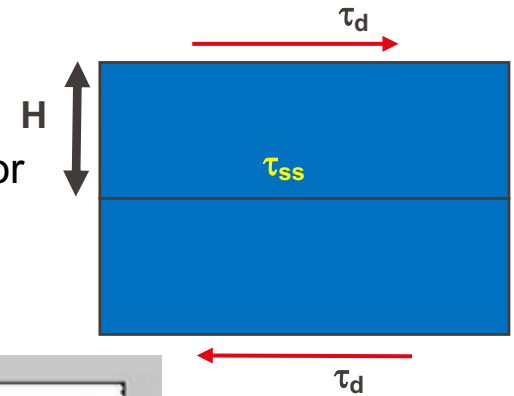


Rezakhani et al., JMPS, 2020

At longer time scales: no stress drop, no crack-like behavior

Friction is friction

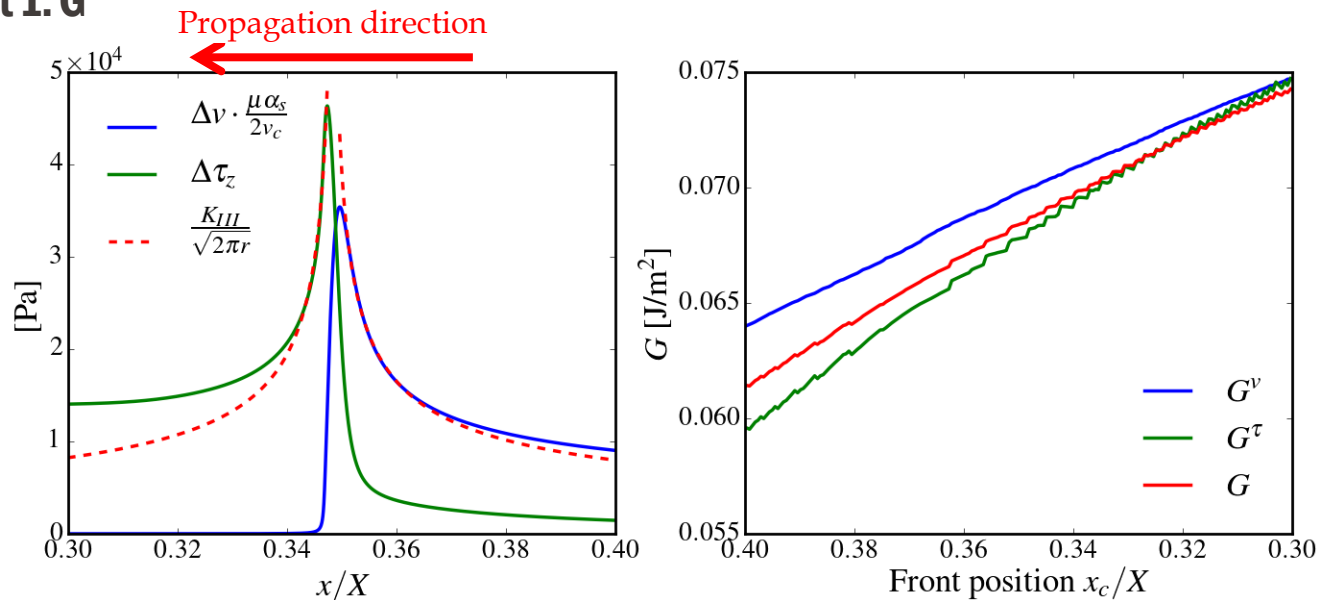
Convergence to equilibrium after several wave reflections



With stress drop: Energy balance of frictional crack

Barras et al., EPSL, 2020

Part 1: G



$$\tau_z = \frac{K_{III}}{\sqrt{2\pi(x - x_{tip})}} + \tau_z^r(t)$$

$$\Delta v = \frac{K_{III}}{\sqrt{2\pi(x - x_{tip})}} \frac{2v_c}{\mu \alpha_s}$$

$$G = \frac{K_{III}^2}{2\mu} A_{III}$$

Energy balance of frictional crack

Part 2: G_c

ϕ : average lifetime of the micro-contacts

Evolution law:

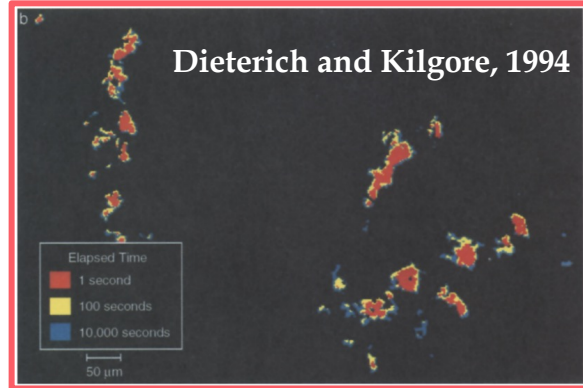
$$\dot{\phi} = 1 - \frac{v\phi}{D}$$

$\frac{v\phi}{D} = 1$: steady-state contact area

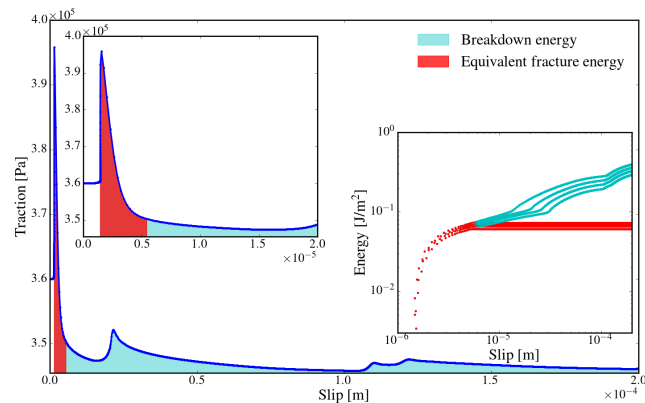
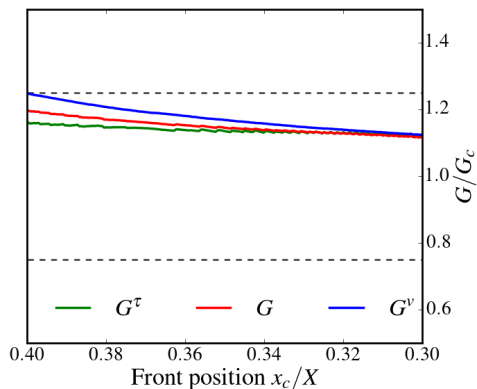
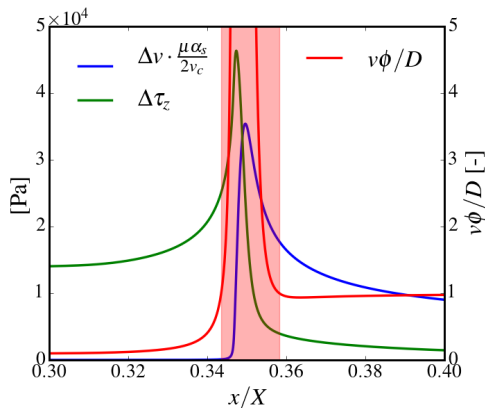
$\frac{v\phi}{D} > 1$: decreasing average contact area

$\frac{v\phi}{D} < 1$: increasing average contact area

$$G_c(t) = \frac{1}{v_c(t)} \int_{\frac{v\phi}{D} > 1} [\tau_z(x, t) - \tau_r(t)] v(x, t) dx$$



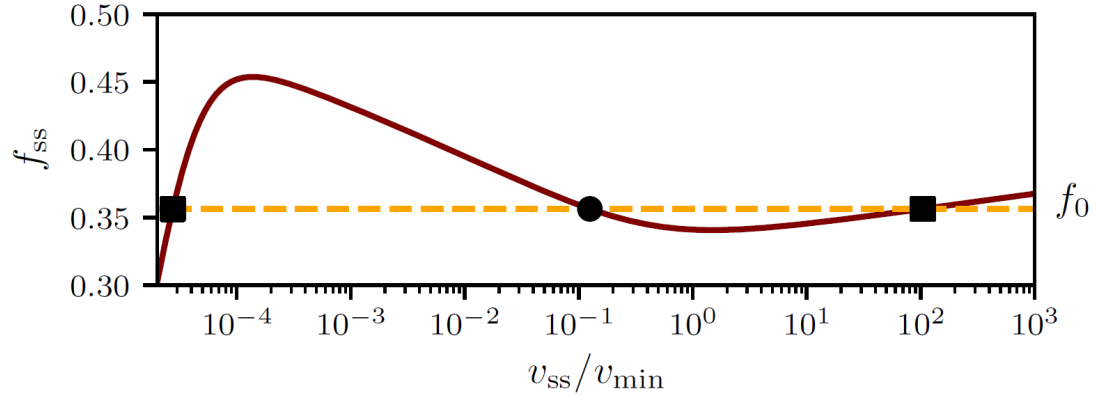
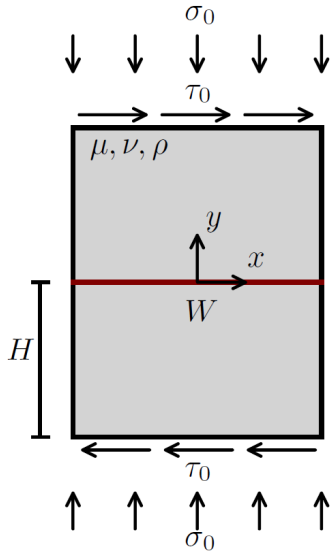
Interaction, Disorder, Elasticity



Short conclusion on frictional cracks

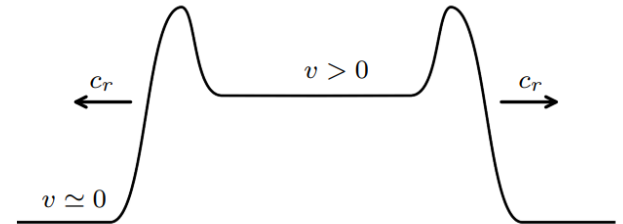
- Finite and well-defined dynamic stress drop is a necessary condition for crack-like behavior
- It is a finite time effect directly related to wave radiation from the interface
- Dynamic stress drop function of interface AND bulk properties
- In presence of stress drop, frictional rupture can be quantitatively described by fracture mechanics energy balance equation
- When no stress drop: friction is friction

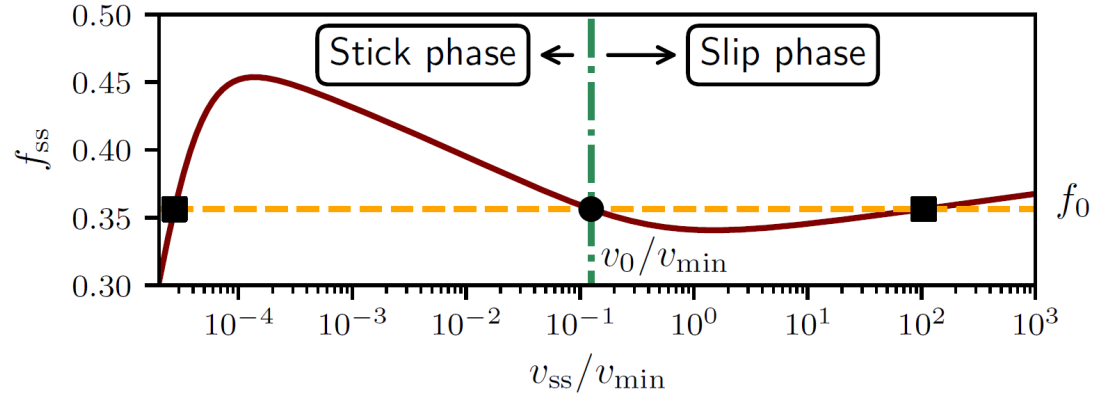
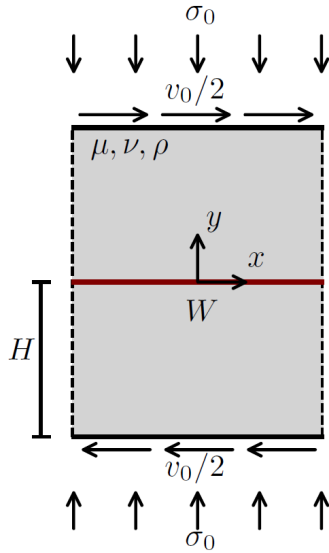
Stress driven frictional rupture



Two stable points:

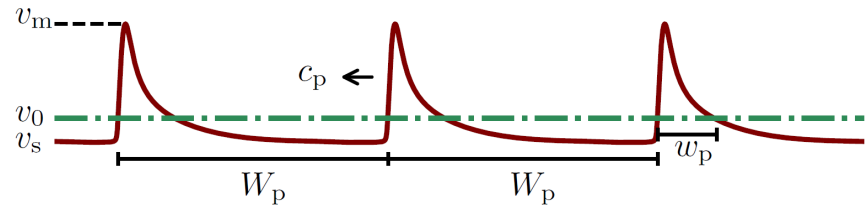
- Initial steady-state
- Post rupture steady-state





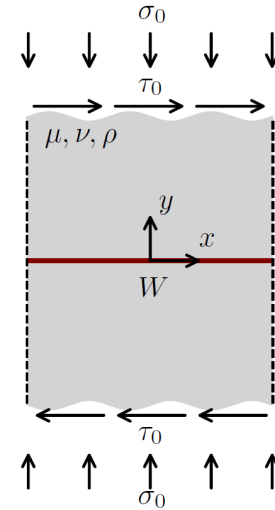
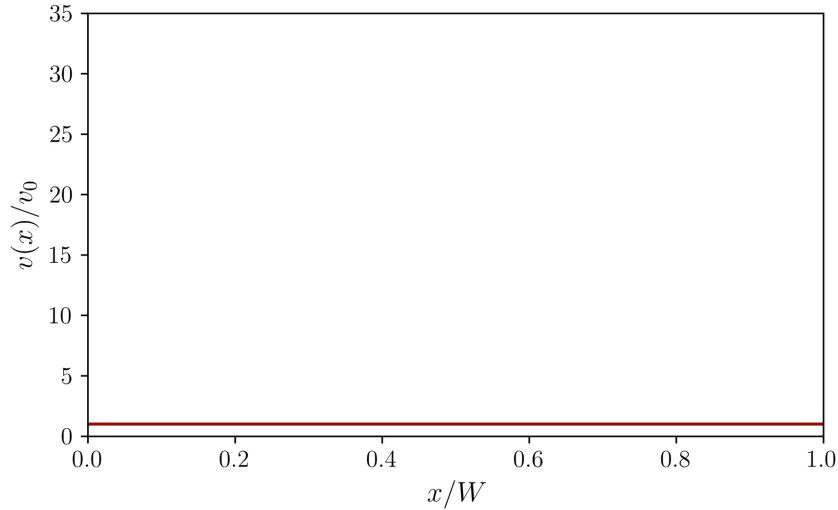
- Single unstable point
- Spatially homogeneous steady-state is impossible

Steady train of pulses



An example with $H \rightarrow \infty$

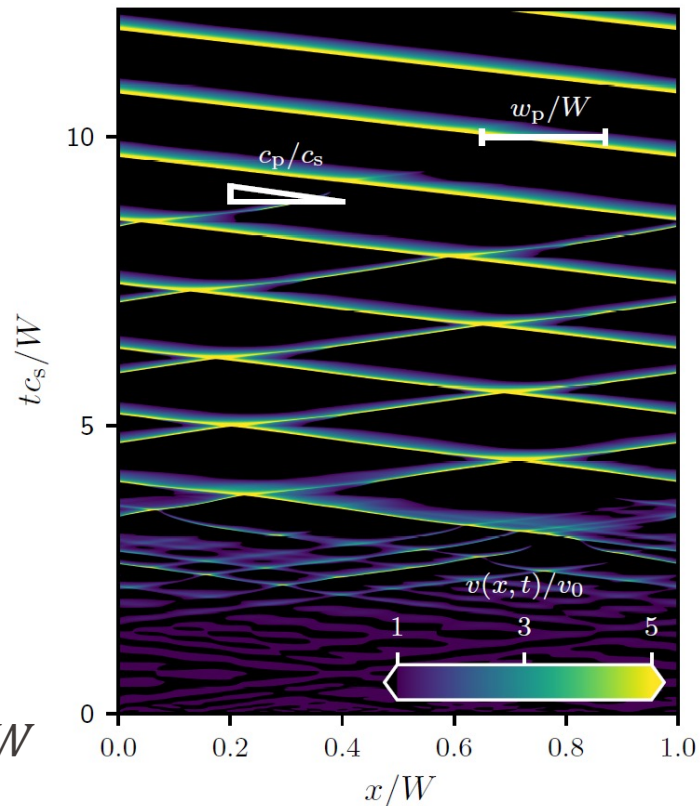
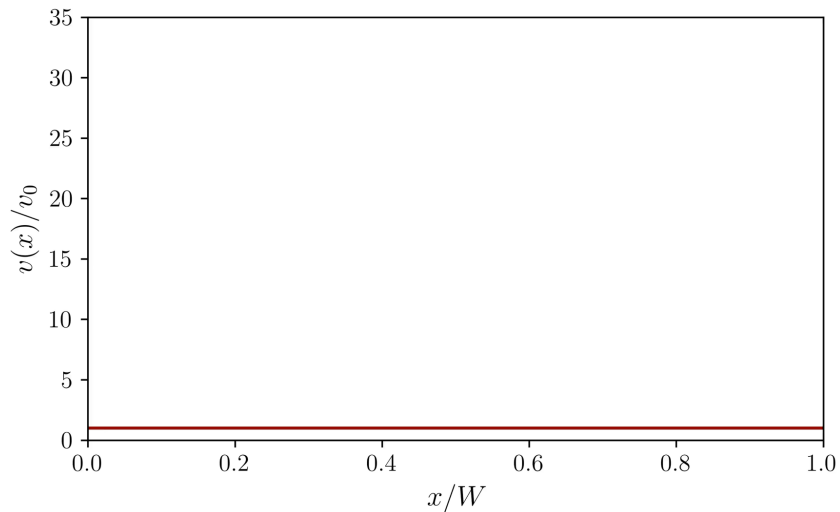
$t = 0.0000 W/c_s$



- SBIM is formulated with stress BC
- Average velocity condition at the interface $\frac{1}{W} \int_0^W v(x) dx = v_0$
- $\tau_0(t)$ is unknown

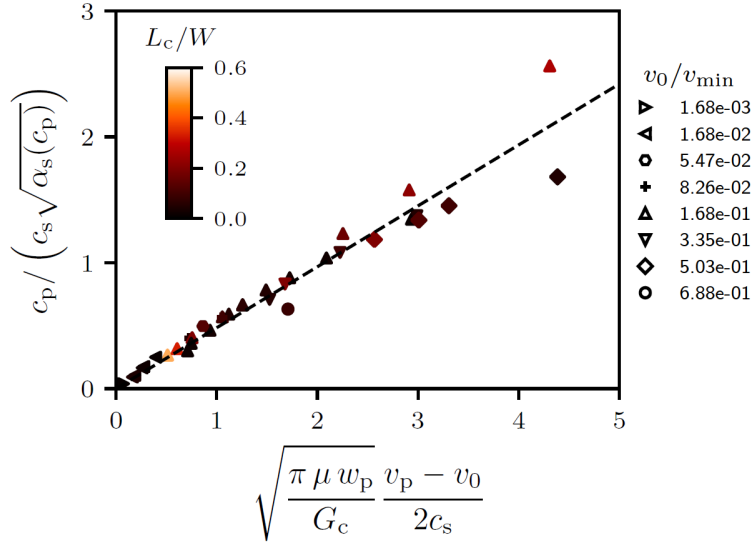
An example with $H \rightarrow \infty$

$$t = 0.0000 W/c_s$$



- Coarsening dynamics
- Single pulse in the periodic domain
- Steady train of pulses of periodicity $W_p = W$

Pulse equation of motion: singular fields and local energy balance



Slip velocity and stress are singular at the tip

$$\tilde{v}(x) \propto \frac{c_p K}{\sqrt{x - x_p}}$$

Assume local energy balance

$$G_c = G = \frac{K^2}{2\alpha_s(c_p)\mu}$$

G_c for R&S friction?

- Fit of singular field
- Analytical estimate

$$c_p = f(v_0, W, G_c, w_p, v_p, \mu)$$

Short conclusion on steady train of pulses

- Velocity-driven frictional give rise to the emergence of steady pulse train
- The coarsening dynamics is saturated at the system width W
- The near rupture fields of slip pulse are singular, implying energy balance and allowing to derive an equation of motion for pulses
- The characteristics of the pulse train are determined by the system size and driving velocity

What if the frictional system does not reach this «long term » steady state?

Can complexity emerge in a system without bulk and frictional heterogeneities ?

Usually, complexity emerges because of disorder in the bulk or frictional properties

What criticality in cellular automata models of earthquakes?

Silvia Castellaro and Francesco Mulargia

Spinodals, scaling, and ergodicity in a threshold model with long-range stress transfer

C. D. Ferguson,^{1,*} W. Klein,¹ and John B. Rundle²

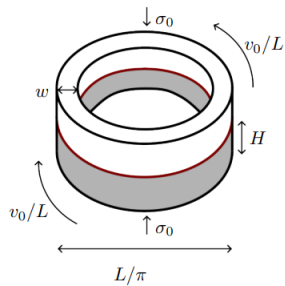
Stochastic properties of static friction

Gabriele Albertini^{a,b}, Simon Karrer^a, Mircea D. Grigoriu^b, David S. Kammer^{a,*}

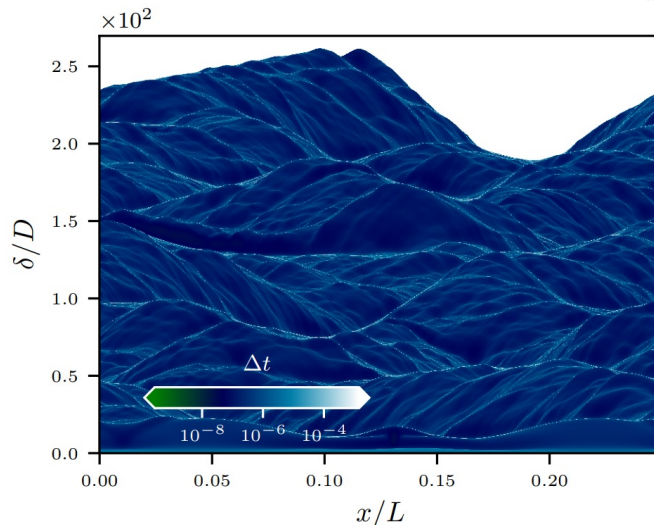
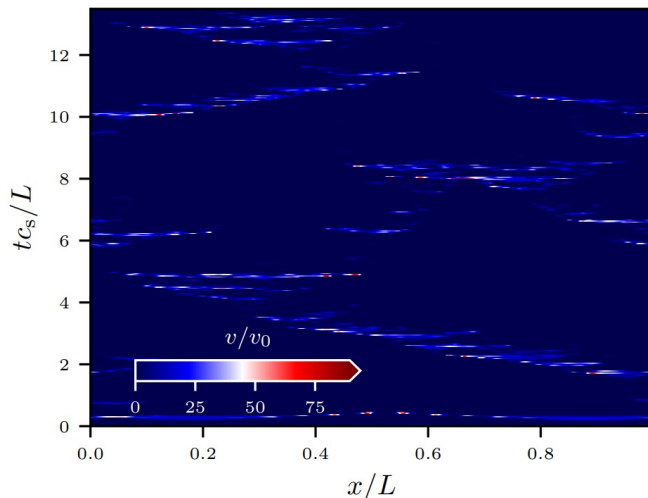
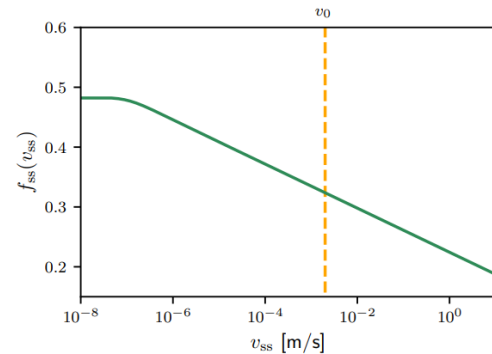
Scaling theory for the statistics of slip at frictional interfaces

T.W.J. de Geus¹, Matthieu Wyart¹

Can complexity emerge in a system without bulk and frictional heterogeneities ?



- Symetric homogeneous elastic bulk
- Homogeneous frictional properties (R&S)
- Finite height H
- FE simulations

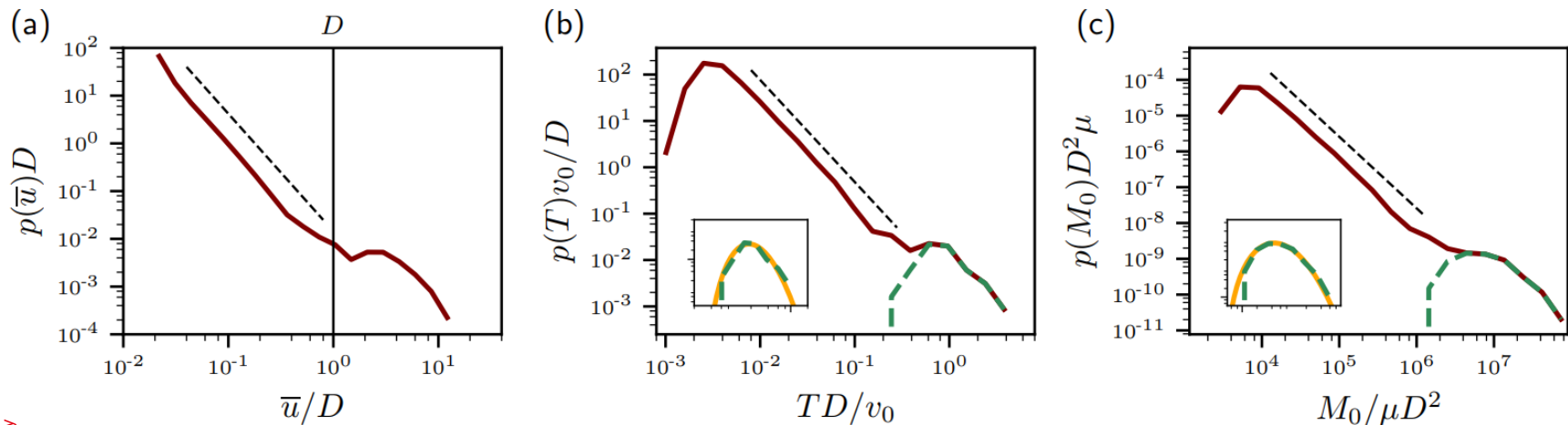


$p(X)$: probability density function

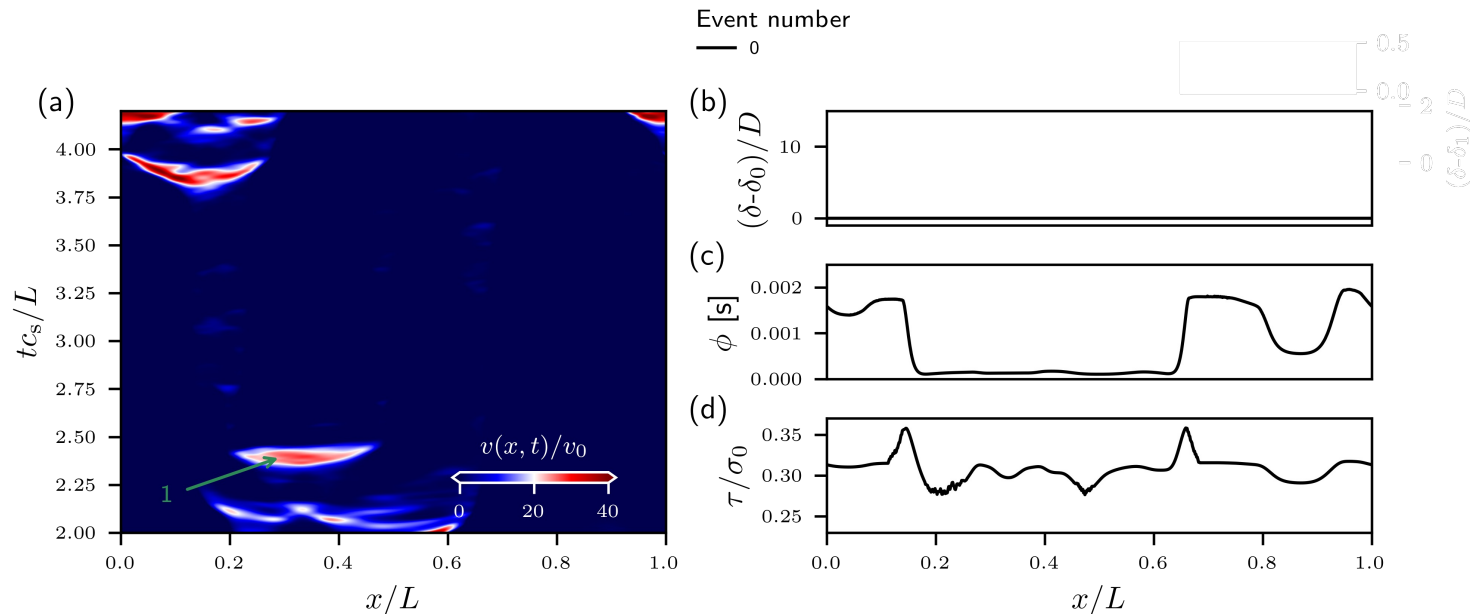
\bar{u} : average slip

T : event duration

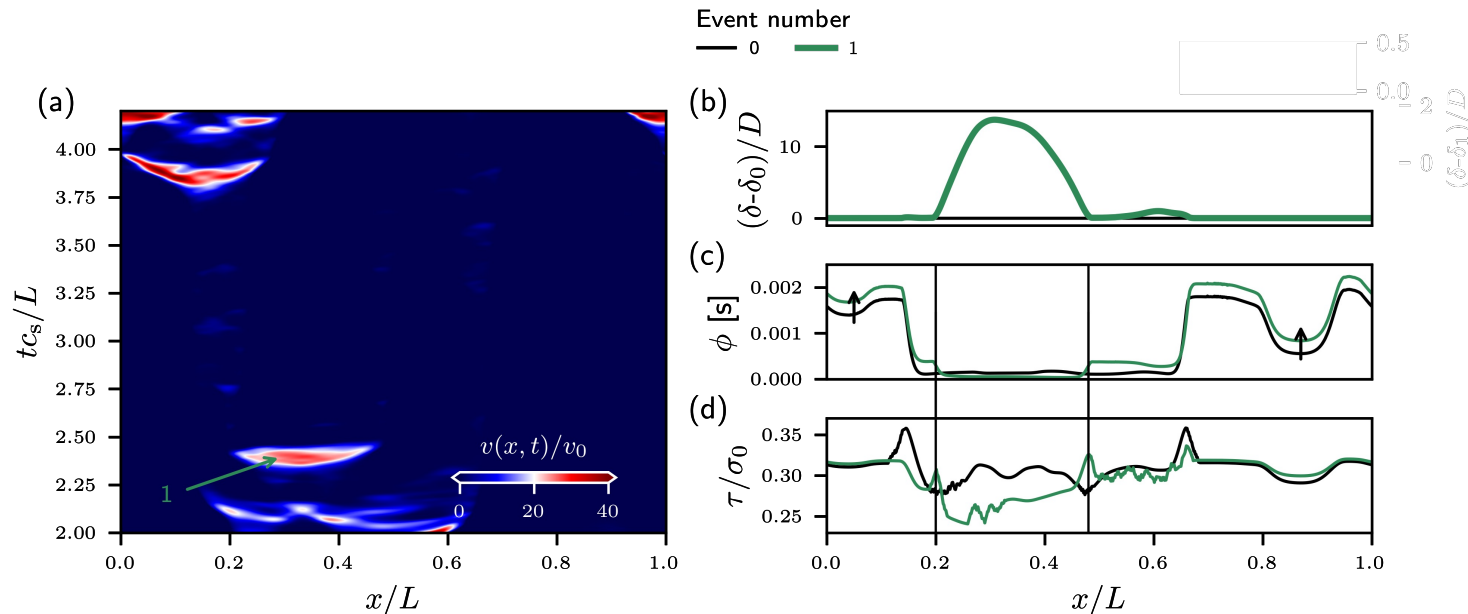
M_0 : seismic moment



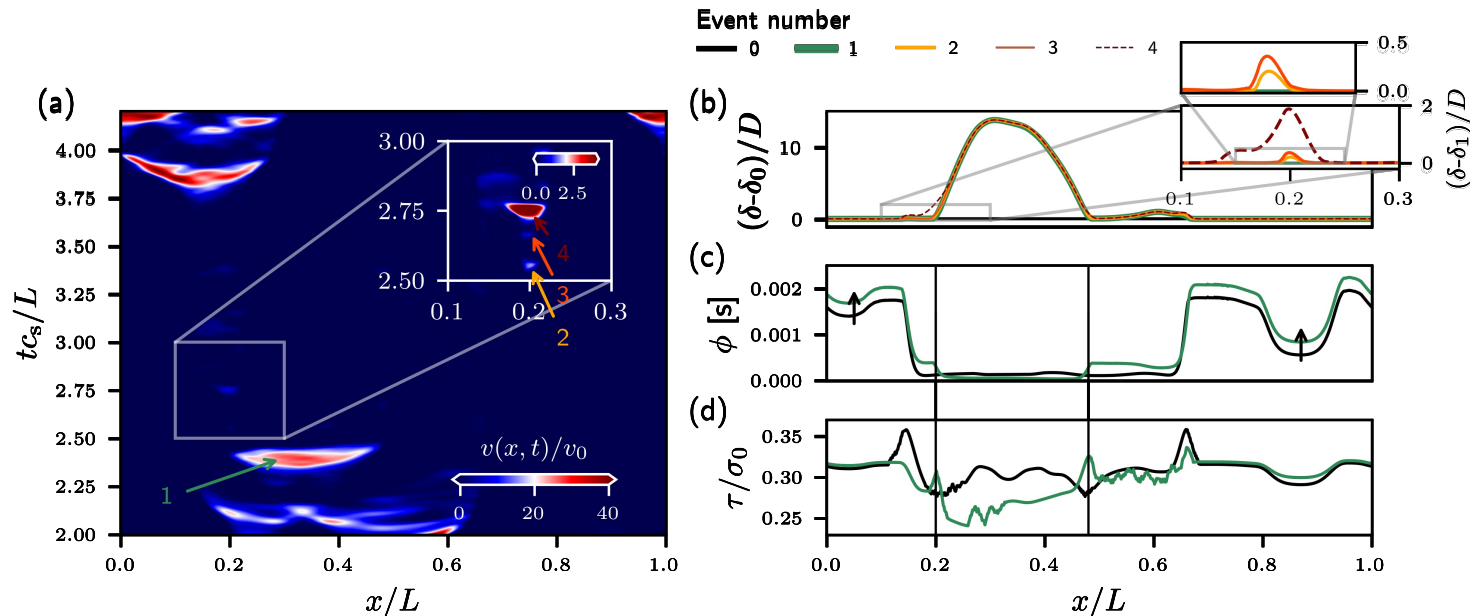
- Broad distribution (several orders of magnitude)
- Two types of events (separation at $\bar{u} = D$)
- Power-law scaling for small / non propagating events
- Log-normal for large / rupture-like events



- Heterogeneous state $\phi(x, t)$ and interfacial stress $\tau(x, t)$
- Related to the history of slip



- Rupture area: decrease of ϕ and τ . Stress concentration at arrest location. Increase of ϕ otherwise (aging)
- Displacement is similar to one of a crack $\delta(x_t) \sim \sqrt{L_r^2 - x_t^2}$
- Rupture stops when propagating into unfavorable stress region

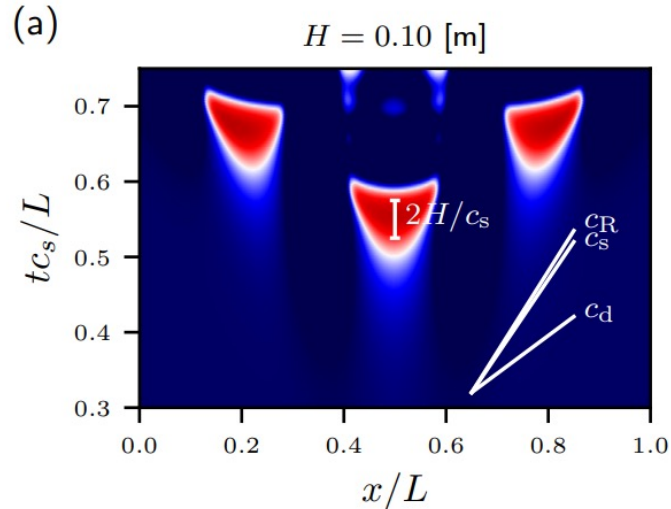


- Small non propagating events $\bar{u} < D$
- Displacement is similar to one of a crack $\delta(x_t) \sim \sqrt{L_r^2 - x_t^2}$
- Occurs at the arrest location of the previous rupture

EPFL The effect of the finite height H on dynamics

Single perturbation analysis

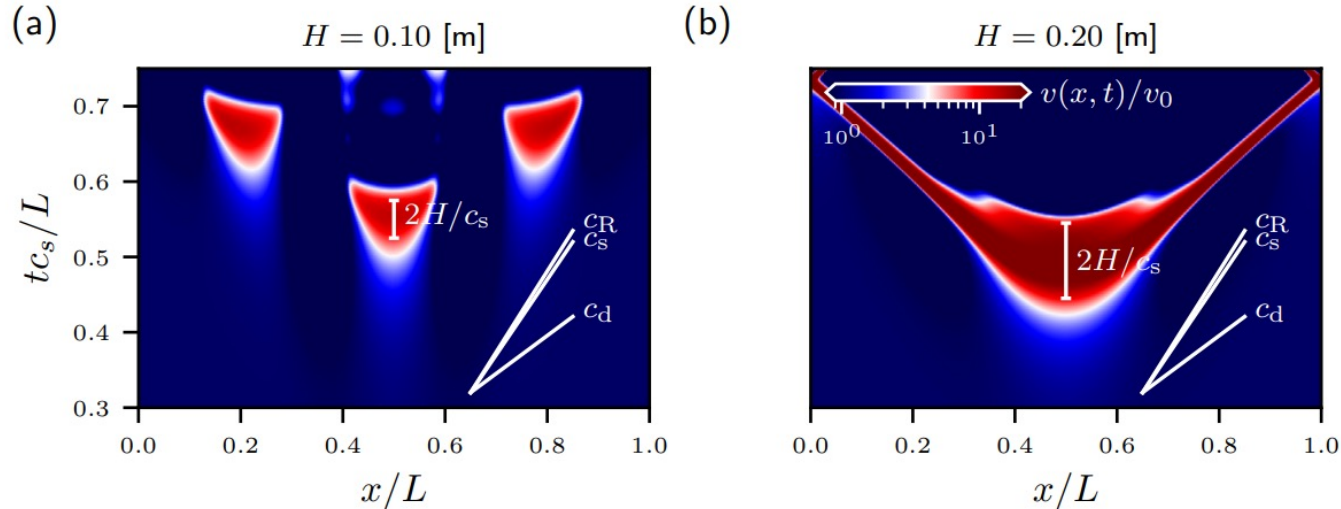
Reflection timescale $2H/c_s$



- Duration of fast slip at $x/L = 0,5$ corresponds to the reflection timescale
- Reflection stops the rupture
- If one doubles H , duration of slip at $x/L = 0,5$ should double as well

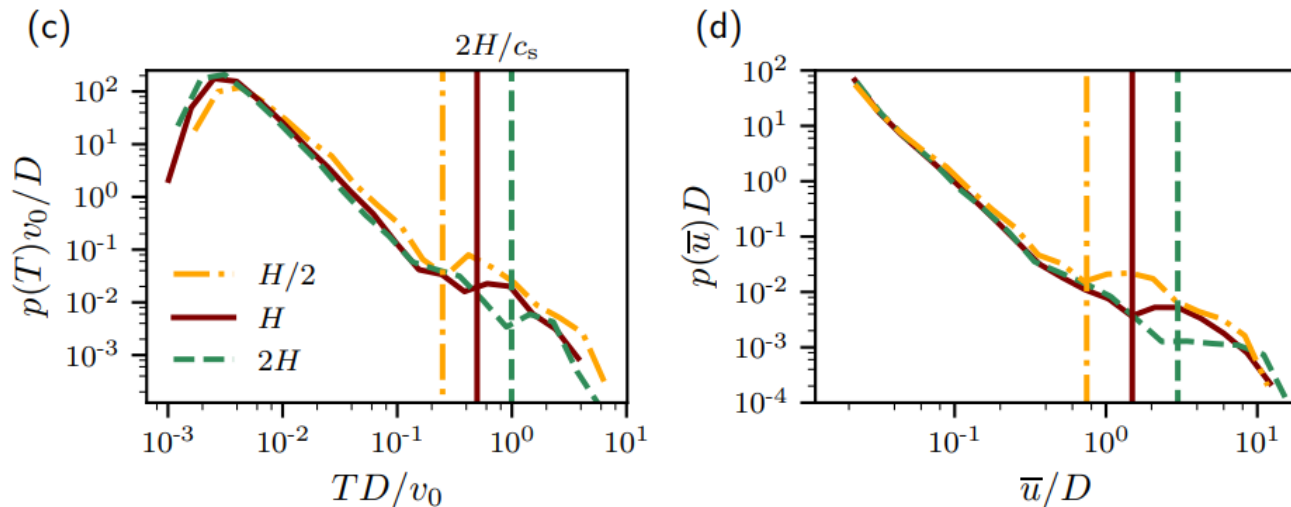
Single perturbation analysis

Reflection timescale $2H/c_s$



- Duration of fast slip at $x/L = 0,5$ has been doubled (still $2H/c_s$)
- Reflection changes a crack like rupture in two slip pulses

Probability distribution of events for various system heights



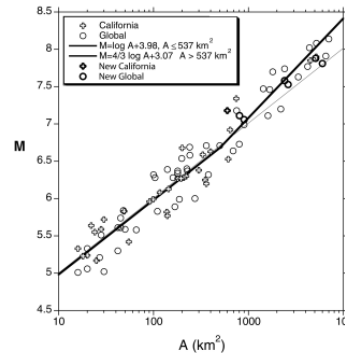
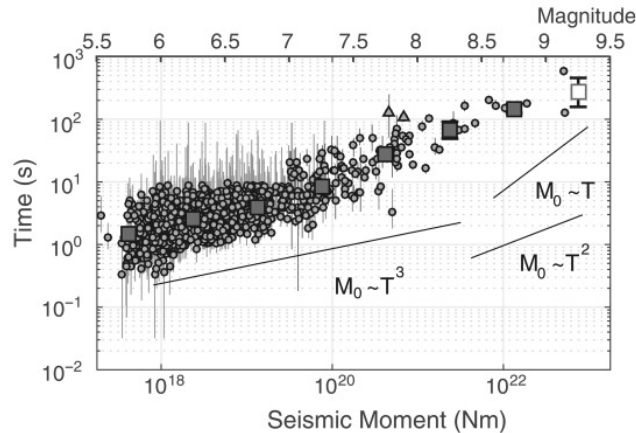
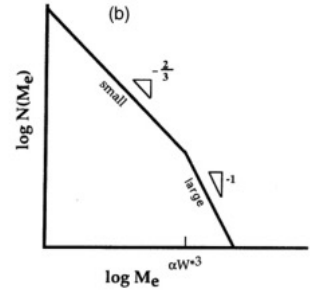
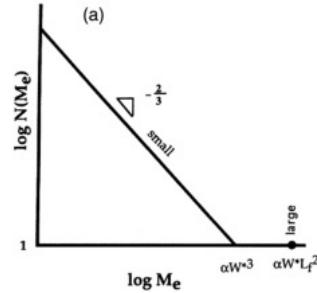
- The change in scaling in the PDF of the duration coincides with the reflection timescale $T_c = 2H/c_s$
- Change in behavior in PDF of slip coincides with a characteristic slip value related to the reflection timescale $u_c = \alpha v_0 2H/c_s$

Relation to statistical complexity in earthquakes ?

- Broad distribution of slip events characteristics, emergence of power-law scaling

- Gutenberg Richter $n(M) \propto M^{1-\beta}$

- Omori's Law $\frac{\Delta N}{\Delta t} = t_0(t + t_1)^{-p}$



The mechanics of earthquakes and faulting, Scholz

- Variety and richness of slip modes at a frictional interface
- Stating the obvious: BCs matter (stress-driven and velocity-driven sliding)
- Crack-like behavior when there is a stress drop (short lived behavior)
- Emergence of train of pulses when displacement controlled sliding
- Emergence of complexity from dynamic elastic interactions with boundaries (Interaction, Disorder, Elasticity)

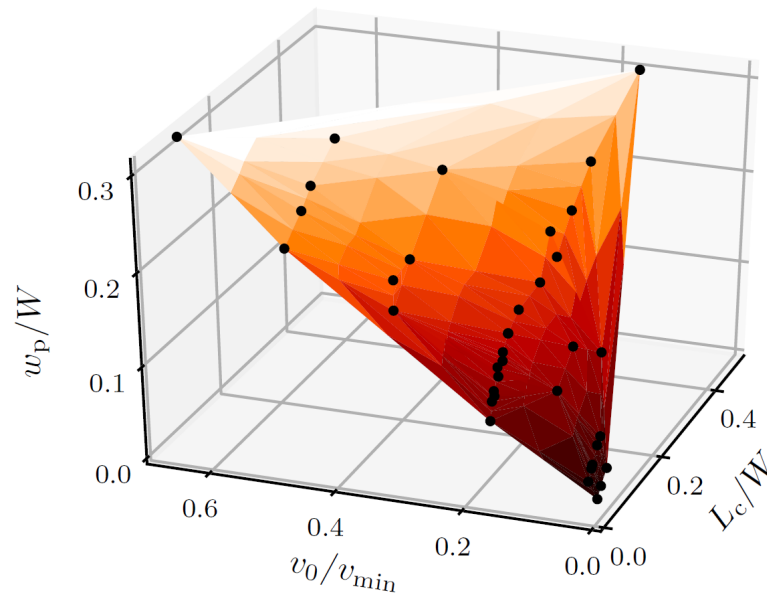
Selection of the pulse width

$$w_p = f\left(\frac{W}{L_c}, \frac{v_0}{v_{min}}\right)$$

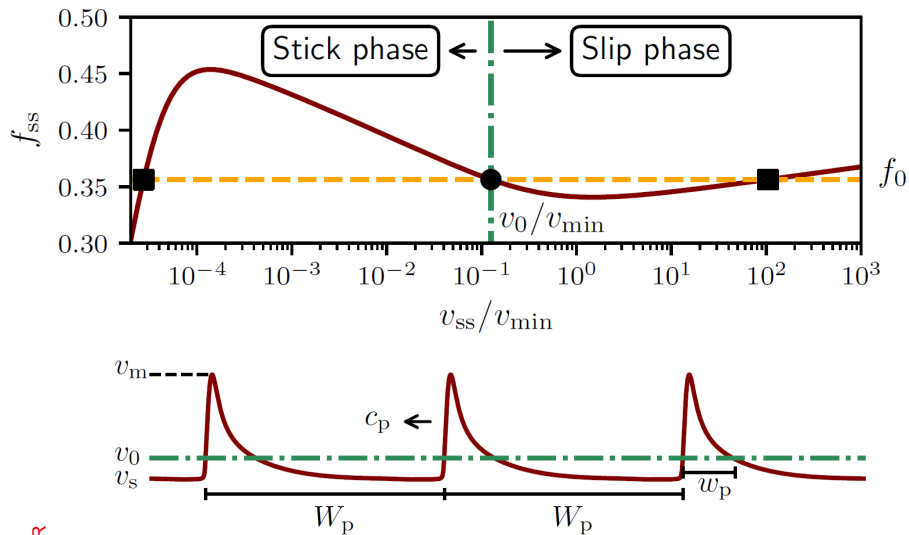
$$v_0 \rightarrow v_{min} \quad : \quad w_p \rightarrow W$$

$$W \rightarrow L_c \quad : \quad w_p \rightarrow W$$

w_p increases monotonically
with both arguments

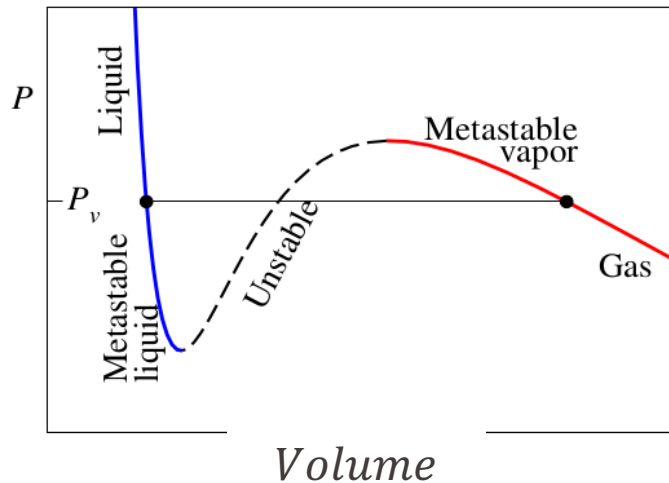


- Velocity weakening (unstable)



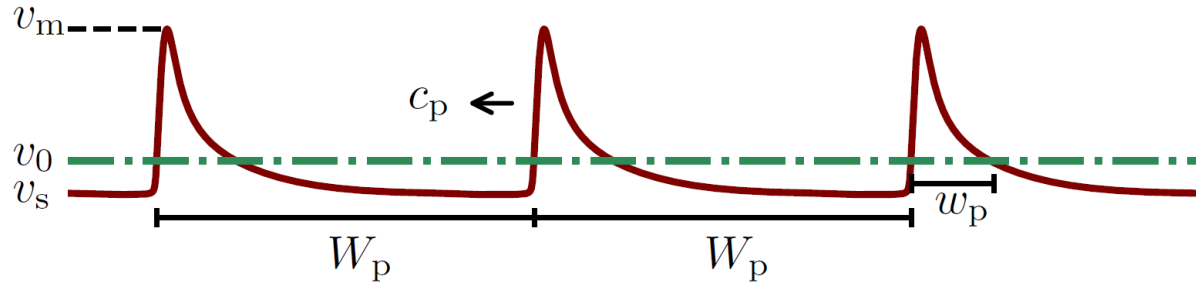
- Two phases (stick and slip)

- P increases with Volume (unstable)



- Two phases (liquid and gas)

How the properties of the pulse train are selected?



Four quantities to fully describe the train

$$W_p \quad w_p \quad c_p \quad v_p$$

$$v_p = \frac{1}{w_p} \int_{w_p} v(x) dx$$

Two equations directly available

$$W_p = W$$

$$w_p(v_p - v_0) - (W - w_p)v_0 = 0$$