History effects in the creep of a disordered brittle material

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Background

- Creep is the slow deformation of a material while subject to persistent mechanical stress

In simulations:

- Instantaneously load the sample to a (subcritical) stress and keep it constant

In experiments:

- Quickly (in a few seconds) ramp up the stress to a (hopefully subcritical) level and keep it constant
  - PID controller

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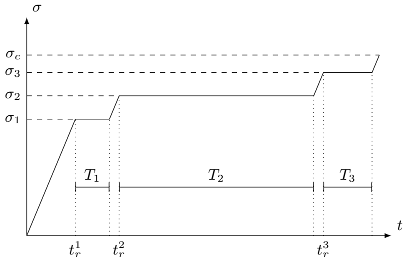
- Stress-stepping means doing a creep experiment at σ_1 for T_1, then increasing the stress to σ_2 for T_2...

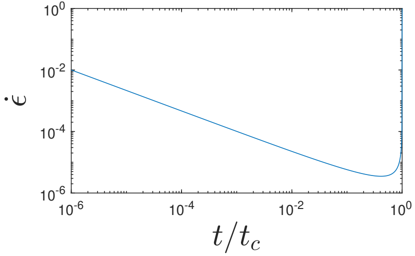


- How do the previous steps influence the creep behavior?

Background – creep

Stages of creep

- Primary creep $\dot{\epsilon} \propto t^{-p}$
  - $p = 2/3$ Andrade creep
  - $p = 1$ logarithmic creep ($\epsilon \propto \log t$)
- Secondary creep $\dot{\epsilon} = \text{const}$
  - Strain rate minima $\dot{\epsilon}_{min}$ achieved
- Tertiary creep $\dot{\epsilon} \propto (t_c - t)^{-\alpha}$
  - Strain rate diverges at failure



- Strain rate also depends on the stress and temperature

$$\dot{\epsilon} \propto \sigma^n \exp\left(-\frac{E}{k_B T}\right) t^{-p}$$

Background – phenomenological approaches

Time-hardening

$$\dot{\epsilon} = \dot{\epsilon}(\sigma, T, t)$$

- The order of the loading steps should not matter

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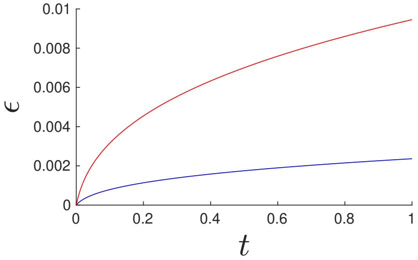

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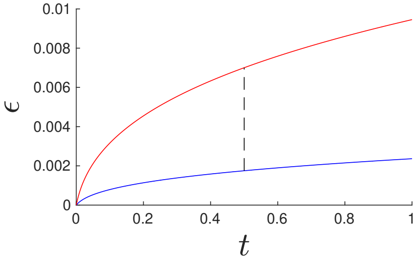

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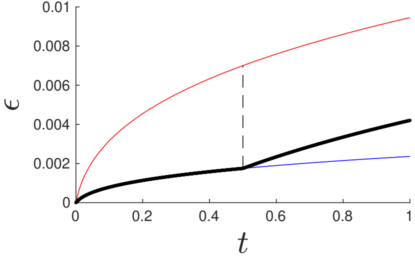

[1] Davenport, J. Appl. Mech. 1938

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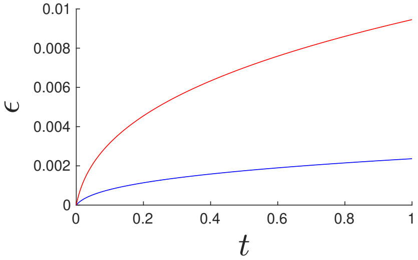

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- Not really useful for brittle materials

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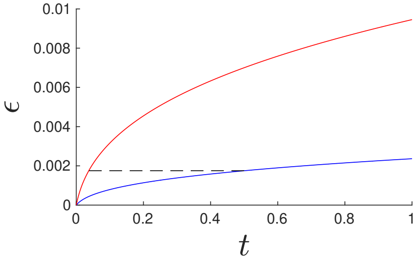

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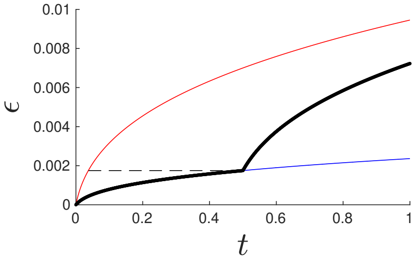

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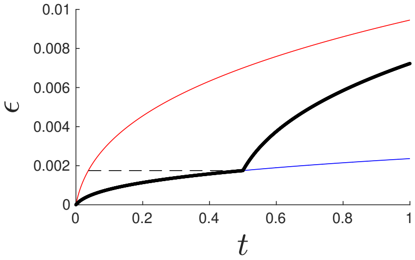
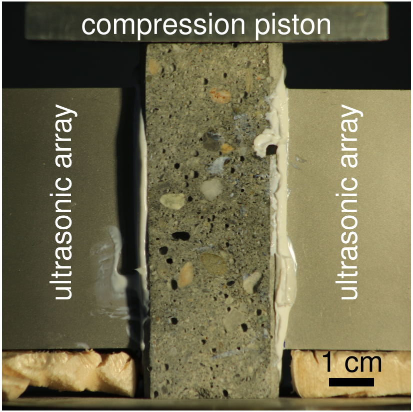


- Physical justification lacking
  - Spoiler: does not work for our data

Experimental setup

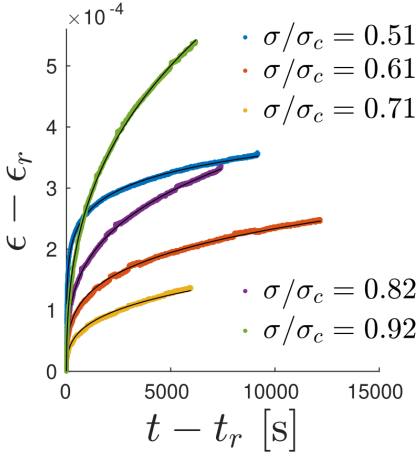
- Creep compression experiments on concrete [2]
- Acoustic emission monitoring
- Ultrasonic tomography [3]
  - ultrasonic pulses sent from one array to another
  - decrease in the amplitude as a sign of damage



[2] Mäkinen et al, PRMaterials 2023
[3] Tudisco et al, J. Acoust. Soc. Am. 2015

Experimental results

- The creep rate is slower when the sample has been previously damaged
  - Increases again close to failure stress
  - Material seems more creep resistant than it actually is
- The *p* exponent also changes
  - Linear decrease with σ



$$\dot{\epsilon} = \frac{K}{(t - t_r + c)^p}$$

Experimental results

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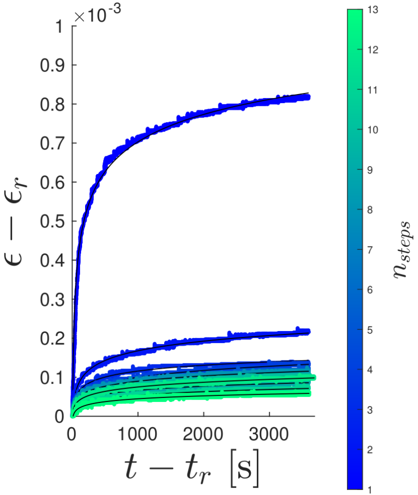
- We did a unloading-reloading test with the same stress for each step



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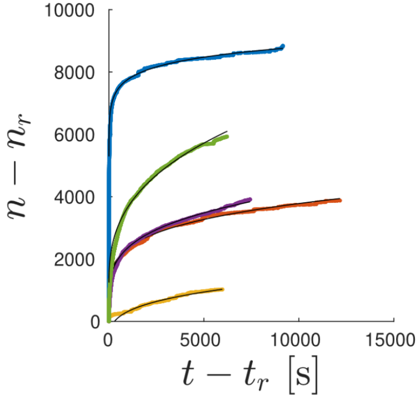
- the number of acoustic events *n* follows brittle creep ($\dot{\epsilon} \propto \dot{n}$)



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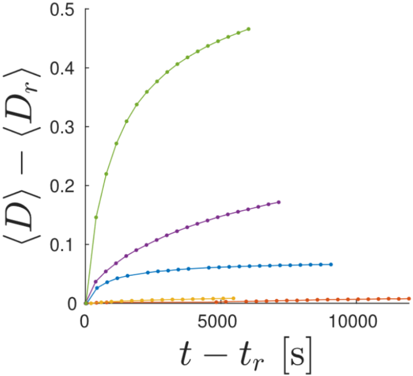
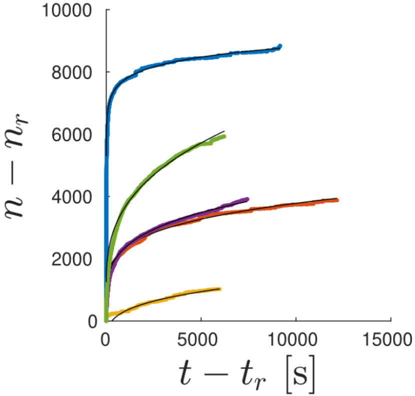


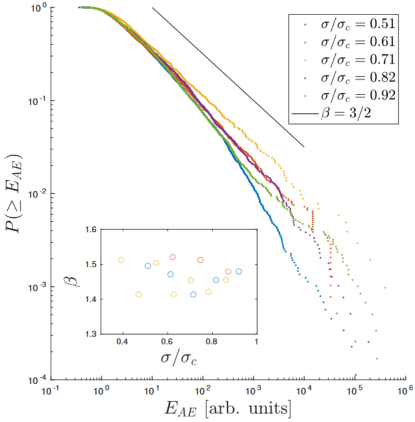


- Ultrasonic tomography shows similar damage behavior

$$D = \frac{A - A_0}{A_0}$$

Experimental results – acoustic emission energies

- No clear cutoff observed in the distributions



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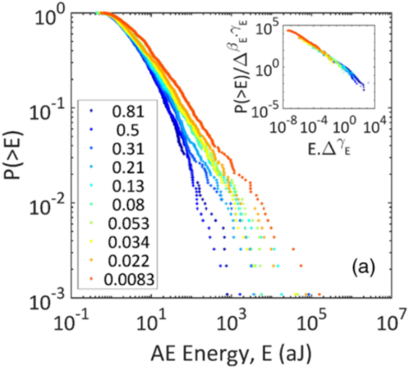
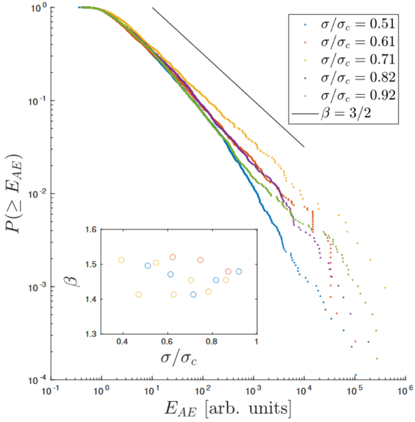


[4] Vu et al, PRL 2019



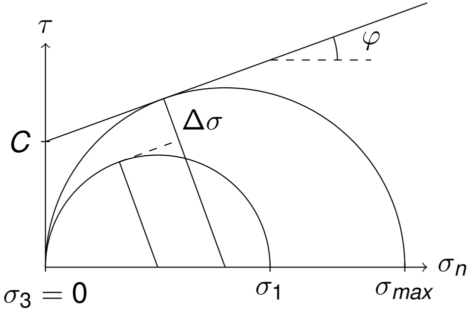
- Differs from monotonic loading

Mohr–Coulomb theory

- Geophysics model for materials which have far better compressive strength than tensile strength

$$\tau = C + \sigma_n \tan\varphi$$

  - τ shear strength
  - C cohesion
  - σ_n normal stress
  - φ angle of internal friction



$$\Delta\sigma = C\cos\varphi + \frac{\sigma_1 + \sigma_3}{2}\sin\varphi - \frac{\sigma_1 - \sigma_3}{2}$$

Kinetic Monte Carlo algorithm

- Sample is comprised of N elementary volumes with activation energies $E_i = \Delta\sigma_i V_a$
  - $\Delta\sigma$ Coulomb stress gap
  - V_a activation volume
- Activation rate given by an Arrhenius expression

$$\nu_i = \nu_0 \exp\left(-\frac{E_i}{k_B T}\right)$$

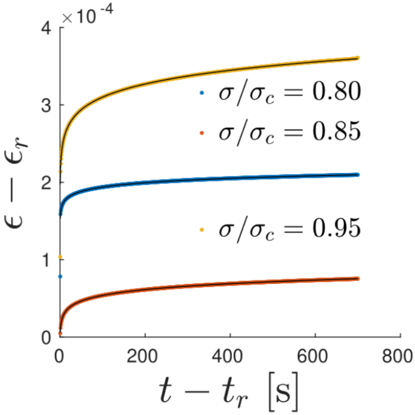
- Each activation damages an element by reducing its Young's modulus by 10 %
- Timestep drawn from distribution

$$p(\Delta t) = \frac{1}{\Delta t_0} \exp\left(-\frac{\Delta t}{\Delta t_0}\right)$$

  - $\Delta t_0 = \left(\sum_i \nu_i\right)^{-1}$
- Cohesion values C_i for each elementary volume are stochastic
  - Microstructural disorder

Simulation results

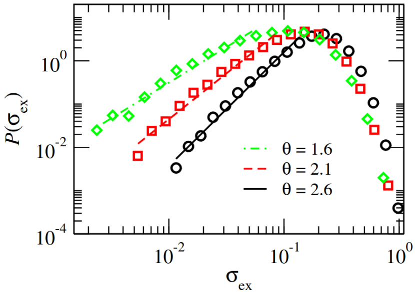
- KMC algorithm with a progressive damage model [5]
- For simplicity we start with a uniform distribution of cohesion values between 0 and C_{max}
- Two step-loading, initial step at $\sigma/\sigma_c = 0.80$
  - Second with 0.85 or 0.95



[5] Amitrano et al, Geophys. Res. Lett. 1999

Excitation spectra

- Stochastic cohesion values give rise to a distribution of Coulomb stress gap values
- We interpret this as the excitation spectra of the sample
  - The distribution of "the distance to damage"

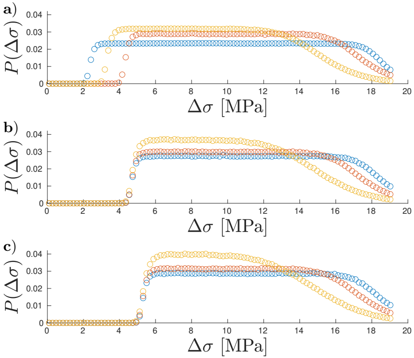


[6] Ovaska et al, PRL 2017

- Power-law behavior at low values

Evolution of the excitation spectra

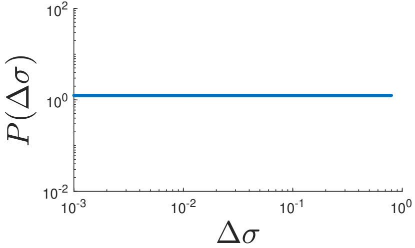
- The small stress gap values (easy-to-damage sites) get depleted
  - Aging-under-stress
- For the $\sigma/\sigma_c = 0.85$ step there is almost no evolution
- Narrowing of the distribution in the $\sigma/\sigma_c = 0.95$ step



	σ/σ_c
a) $t = 0.35$ s	Blue: 0.80
b) $t = 10$ s	Red: 0.85
c) $t = 700$ s	Yellow: 0.95

Evolution of the excitation spectra – schematic

- From an initial distribution the shape narrows during primary creep as the small values are exhausted
- Secondary creep corresponds to a very narrow distribution
- In tertiary creep the distribution widens again
  - General softening due to damage accumulation



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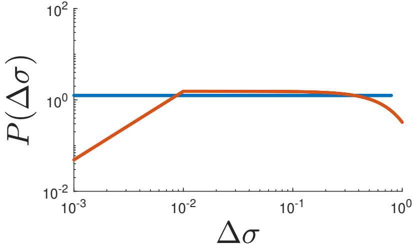

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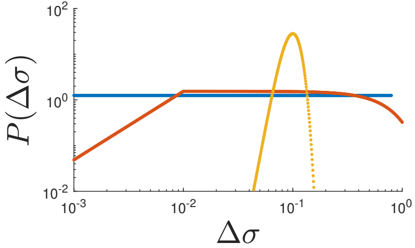

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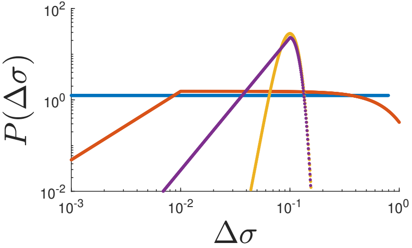

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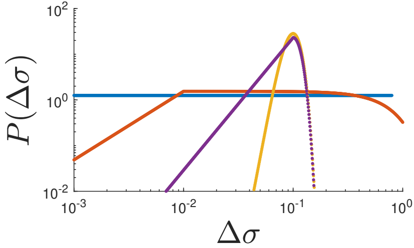
[7] Cottrell, J. Mech. Phys. Solids 1952



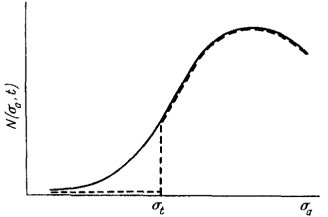


Fig. 2. The distribution of elements with activation energies in a given range : full curve – start of creep ; broken curve – after creep for a time t.

Summary

- We observe loading history effects in the creep of a disordered brittle material (concrete)
- The behavior is more complex than the phenomenological models in the literature suggest
- We interpret this as an *aging-under-stress* phenomenon where the easy-to-damage sites are exhausted
- The acoustic emission behavior does not show a clear cutoff
  - differs from monotonic loading
- the work is continued [8] in collaboration with the previous authors and Mikko Alava
  - go see the poster of Juan Carlos Verano Espitia

[8] Weiss & Amitrano, PRMaterials 2023

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